Telekom-Control Commission
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Rules of Procedure under § 55 Par. 9 TKG 2003
Auction Procedure for Frequency Assignments in the 800 MHz, 900 MHz and 1800 MHz Bands
(non-binding Web version)

NON-BINDING TRANSLATION

1. General

Paired frequency blocks in the 800 MHz, 900 MHz and 1800 MHz bands will be auctioned off in several categories each. The auction procedure will be carried out in the form of a combinatorial clock auction.

The procedure consists of:

- a pre-auction, in which the frequencies reserved for potential new entrants in the 800 MHz band are awarded as an abstract frequency block; and
- a main auction, consisting of a principal stage during which the number of frequency blocks is determined that each successful bidder will receive in each of the categories available to all bidders; and an assignment stage in which the specific frequency blocks are assigned to the winning bidders of the pre-auction and the principal stage.

The frequency blocks in the 1800 MHz band will be allocated separately for the period from 2013 to the end of 2019 (referred to as the transition period) and from 2020 until the end of the licensing term.

The pre-auction and the principal stage of the main auction each consist of a number of open rounds of bidding (clock stage) and a sealed-bid stage.

- The bidders admitted to the pre-auction can indicate in each of the open-bidding rounds whether they wish to acquire the frequencies reserved for new entrants at the current price. They can then submit a final bid for those frequencies in the sealed-bid stage. The bidder with the highest bid wins the auction. The price to be paid by the winning bidder will be determined by the second-highest bid, or where only one bidder participates in the pre-auction, the minimum bid.
- In the principal stage (or allocation stage) of the main auction the bidders can submit a combinatorial package bid for frequency blocks by specifying the number of (abstract) frequency blocks in each category that they wish to acquire at the current price. Once the clock stage has been completed, the bidders may submit supplementary package bids for (other) combinations of abstract frequency blocks in the sealed-bid stage (or supplementary stage). The winning combination of bids which maximises the revenue will then be determined algorithmically from all bids submitted during the principal stage; at most one bid from each bidder (from all bids submitted during the clock stage and the sealed-bid stage) will be included in the winning combination. The winning bidders will be those whose bids are included in the winning combination (maximising the total auction revenues) of package bids. The winning bidders will receive the number of frequency blocks indicated in their winning bids in each category at the respective “base prices”. These base prices will be determined on the basis of a modified second-price rule and are the lowest prices the winning bidders (together) would have had to offer in order to still be successful (minimum core prices). Moreover, the base price of each winning bid must be at least as high as the sum of minimum bids (start bid or minimum bids) for the frequency blocks included in the respective package.

The second stage of the main auction (the assignment stage) will consist of two sealed-bid rounds of bidding in which bidders can submit package bids on various combinations of specific frequency blocks which are compatible with the number of frequency blocks won by the successful bidders in the principal stage or in the pre-auction.

- In the first bidding round, the frequency blocks in the 1800 MHz band will be assigned for the period from 2013 to the end of 2019. As some pre-existing licences for these frequencies will not have expired at the time of the auction, some of those frequency

blocks will not be (fully) available for use during that period. In category C2 the frequency ranges assigned will not necessarily be contiguous. In the other two categories, contiguous spectrums will be assigned.

- In the second bidding round, the contiguous spectrums in the 800 MHz and the 900 MHz bands (where necessary) and in the 1800 MHz band (categories C1 to C3) will be simultaneously assigned for the period from 2020. For the purpose of assigning the frequency blocks in the 1800 MHz band for the period starting in 2020, the blocks won in the categories C1 to C3 will be added up for each bidder in the principal stage, i.e. the categories C1 to C3 will be combined into one single category C, in which contiguous spectrums are assigned (starting from 2020).

The winning bidders will receive the specific frequency blocks as indicated in their successful bids in each category at the respective "top-up" prices. The top-up prices will also be determined on the basis of a modified second-price rule. Before the second bidding round, the bidders will be informed of the result of the first round, i.e. the assignment of the frequency blocks for the transition phase.

The total price to be paid by each winning bidder results from the sum of the prices of the pre-auction, the principal stage of the main auction and the assignment stage of the main auction.

Those applicants will be admitted to the pre-auction and the principal stage of the main auction that have not been excluded from the frequency assignment procedure pursuant to Art. 55 Par. 8 No. 2 TKG 2003 and, if applicable, meet the TKK requirements and qualify as a new entrant (cf. Section 2.2 of the Tender Documentation).

Those applicants will be admitted to the assignment stage of the main auction that were successful in the pre-auction or the principal stage of the main auction and have acquired abstract frequency blocks for which more than one assignment option is available.

The maximum amount of spectrum which a bidder can acquire is limited by the eligibility requested by that bidder, and by the spectrum caps defined by the Telekom-Control Commission (cf. Sections 4.3 and 4.4 of the Tender Documentation).

The auctioneer will be the Telekom-Control Commission or a member appointed by the Commission. The Telekom-Control Commission may also appoint employees of RTR's Telecommunications Division to carry out the auction.

2. Collusion and discontinuation of the procedure

2.1. Collusion

Any and all forms of cooperation among the applicants or their stakeholders, be it direct or indirect, with the intention of influencing the events or results of the auction (collusive behaviour) are prohibited. Should applicants cooperate in a collusive manner before or during the auction procedure, they may be excluded from the remainder of the procedure (cf. Section 2.5 of the Tender Documentation). The auctioneer has the right to take all appropriate measures to prevent collusive behaviour.

Similarly, any threatening of competitors or making public participation in the auction, bid amounts or bidding strategies, even prior to the auction procedure, may result in the applicant's exclusion from the procedure.

In this context, specific reference is also made to the provisions of general competition law in Austria and Art. 168b of the Austrian Penal Code (StGB).

2.2. Discontinuation of the procedure

The Telekom-Control Commission has the right to discontinue the auction procedure if they discover collusive behaviour among applicants and an efficient, fair and non-discriminatory procedure cannot be guaranteed (Art. 55 Par. 12 No. 1 TKG 2003), or if other significant reasons are found which endanger the proper execution of the frequency assignment procedure. Under these circumstances, the Telekom-Control Commission will decide whether the procedure is to be discontinued in accordance with Section 2.6 of the Tender Documentation or a new auction date is to be set.

3. Objects of the auction

3.1. Eligibility to participate

3.1.1. Those applicants that are not excluded from the frequency assignment procedure in accordance with Art. 55 Par. 8 TKG 2003 will be admitted to the auction. Only those applicants will be admitted to the pre-auction that meet the relevant TKK requirements and qualify as new entrants (cf. Section 2.2 of the Tender Documentation).

3.2. Objects of the auction

3.2.1. The following frequency blocks will be auctioned off (cf. Section 3 of the Tender Documentation):

- Category A1: one specific frequency block of 2x5 MHz in the 800 MHz range;
- Category A2: four abstract frequency blocks of 2x5 MHz each in the 800 MHz range;
- Category A3: one abstract frequency block of 2x5 MHz in the 800 MHz range; to this block, increased coverage requirements apply (cf. Section 3.4 of the Tender Documentation);
- Category B1: one specific frequency block of 2x5 MHz in the 900 MHz range;
- Category B2: five abstract frequency blocks of 2x5 MHz each in the 900 MHz range;
- Category B3: one specific frequency block of 2x5 MHz in the 900 MHz range;
- Category C1: two specific frequency blocks of 2x5 MHz each in the 1800 MHz range;
- Category C2: eight abstract frequency blocks of 2x5 MHz each in the 1800 MHz range;
- Category C3: five abstract frequency blocks of 2x5 MHz each in the 1800 MHz range;

Two frequency blocks of the A2 category are reserved for a new entrant and will be auctioned off in the pre-auction as one abstract frequency block. If none of the bidders is successful in the pre-auction, those frequency blocks will be auctioned off in the principal stage of the main auction.

Each frequency package is assigned a certain number of eligibility points (bidding points). Table 1 below provides an overview of the frequency blocks, their respective designations in the auction procedure, and the number of eligibility points associated with each package.

Table 1: Overview of frequency blocks to be auctioned-off

Category	Frequency range	Block designation	Eligibility points
A1	800 MHz	LA01	Two eligibility points per frequency block
A2 and A3	800 MHz	LA02	
	800 MHz	LA03	
	800 MHz	LA04	
	800 MHz	LA05	
	800 MHz	LA06	
B1	900 MHz	LB01	
B2	900 MHz	LB02	
	900 MHz	LB03	
	900 MHz	LB04	
	900 MHz	LB05	
	900 MHz	LB06	
B3	900 MHz	LB07	
C1	1800 MHz	LC01	One eligibility point per frequency block
	1800 MHz	LC02	
C2	1800 MHz	LC03	
	1800 MHz	LC04	
	1800 MHz	LC05	
	1800 MHz	LC06	
	1800 MHz	LC07	
	1800 MHz	LC08	
	1800 MHz	LC09	
	1800 MHz	LC10	
C3	1800 MHz	LC11	
	1800 MHz	LC12	
	1800 MHz	LC13	
	1800 MHz	LC14	
	1800 MHz	LC15	

3.2.2. For the first round of the principal stage, the round prices for each category according to Section 4.2 of the Tender Documentation will be set equal to the minimum bids for one frequency block in each of the categories as shown in Table 2 below

Table 2: Minimum bids for frequency blocks to be auctioned off

Category	Minimum bid in EUR
A1	32,000,000
A2	32,000,000
A3	32,000,000
B1	23,400,000
B2	29,900,000
B3	23,400,000
C1	14,600,000
C2	8,800,000
C3	11,400,000

3.2.3. The minimum bid for the frequency block offered in the pre-auction is EUR 45,600,000.

4. Pre-auction

4.1. General

4.1.1. The pre-auction will only take place if at least one bidder has applied for participating in the pre-auction according to Section 5.2.1 of the Tender Documentation and complies with the admission requirements.

4.1.2. If no bidder is admitted to the pre-auction, the auction will begin immediately with the principal stage of the main auction.

4.2. Bids and submission of bids

4.2.1. In the pre-auction, the bidders admitted to participate will submit bids for the spectrum reserved for a new entrant. The bid constitutes a binding offer of a bidder to pay, as a maximum, the quoted amount for the reserved spectrum.

4.2.2. The bid amount in the open rounds of bidding will be determined by the price set by the auctioneer for the given round. This price is set by the auctioneer before the start of each round of bidding (cf. Rule 5.7).

4.2.3. In the first round of the clock phase the price will be set equal to the minimum bid amount. In the following rounds of the clock phase, the price will be increased if excess demand exists, i.e. more than one bidder has submitted bids for the reserved spectrum. No price reduction will take place.

4.2.4. The amount by which the round price is increased will be determined by the auctioneer. The price increase will generally be between 2% and 15%, but the

auctioneer reserves the right to apply different price increments. All prices will be set in whole euro amounts.

4.2.5. The clock stage ends:

- if no more than one bid is submitted for the reserved spectrum; or
- at the auctioneer's decision, even if more than one bid was received for the reserved spectrum.

The auctioneer will end the clock stage only if convinced that an immediate transition to the sealed-bid stage is in the general interest of an efficient frequency assignment procedure.

4.2.6. In the sealed-bid stage, bidders that were active in the last round of the clock stage may freely choose the amount of their bids. The bids submitted must be in whole euro amounts and must not be below the minimum bid. Bidders that were no longer active in the last round of the clock stage may increase their bids, but in that case the bid cannot exceed the price in the last clock round in which the bidder would have been able to submit a bid.

4.2.7. Bids will be submitted electronically using the auction software (cf. Section 7). The submission of bids by telephone will only be permitted in exceptional cases (e.g. technical difficulties). The auctioneer will be responsible for deciding whether such an exception is warranted.

4.2.8. The amount of each bid submitted during the pre-auction must comply with the rules regarding the bank guarantee as specified in Section 5.3.5 of the Tender Documentation. Bids which exceed the bidding limit arising from the respective bank guarantee will not be considered valid.

4.3. Activity rule

4.3.1. Bidders that submit no bid or a "zero bid" in the clock round will be excluded from further participation in the clock stage. However, this does not affect the right under Rule 4.2.6 to submit a sealed bid after completion of the clock stage.

4.4. Round extensions

4.4.1. At the beginning of the pre-auction, each bidder will be entitled to one round extension.

4.4.2. An extension provides a bidder with additional time to submit a bid during the clock stage. If one of the bidders is entitled to submit a bid but fails to do so during the time allotted, the software will extend the clock round automatically by another 30 minutes and that bidder's right of extension will be subtracted. Round extensions will only be subtracted from those bidders that do not manage to submit a bid during the time allotted for that clock round.

4.4.3. Bidders that do submit a bid during the time allotted for the round will not be able to revise their bids during the additional (extension) time. Bidders that have no more round extensions and do not manage to submit a bid during the time allotted for the round will not be allowed to submit a bid during an extension. The auction software will automatically enter a zero bid for those bidders.

4.4.4. Rounds in which one or more bidders make use of a round extension will come to an end no later than either 30 minutes after the originally scheduled end time, or once all bidders that used a round extension during that round have successfully entered a

bid, whichever is shorter. The auction software will automatically enter a zero bid for bidders that exercise their extension right and fail to submit a bid during the round extension period (see Rule 4.4.3).

4.5. Valid bids

4.5.1. A bid will be considered valid only if submitted within the time period defined by the auctioneer for the given round or during the extended time allotted to the bidder exercising a round extension right (cf. Rule 4.4.4). This also applies to the submission of bids by telephone.

4.6. Determination of winning bidders and prices

4.6.1. After the end of the sealed-bid stage, the auctioneer will determine the bid with the highest bid amount from among all bids submitted during the clock stage and the sealed-bid stage.

4.6.2. The bidder submitting the highest bid wins the auction. If two or more bidders have submitted the highest bids in the same amount, the decision will be taken by drawing lots.[1]

4.6.3. The price to be paid by the winning bidder is

- the second-highest bid amount in the event that several bidders have submitted bids for the reserved spectrum; or
- otherwise the minimum bid.

4.7. Information released to bidders

4.7.1. At the beginning of each round in the clock stage, the auctioneer will release the following minimum information to each bidder admitted to the pre-auction:

- the current round;
- the time at which the round ends;
- the price for the reserved spectrum;
- the remaining number of round extensions held by the bidder.

4.7.2. After the end of each round in the clock stage, the auctioneer will release the following minimum information to each bidder admitted to the pre-auction:

- the bid submitted by that bidder;
- the remaining number of extensions held by the bidder.

4.7.3. The auctioneer will inform each bidder admitted to the pre-auction accordingly once the clock stage has ended (cf. Rule 4.2.5).

4.7.4. At the beginning of the sealed-bid stage the auctioneer will inform each bidder admitted to the pre-auction at least of the time when the sealed-bid stage ends.

4.7.5. After the end of the pre-auction the auctioneer will inform each participant whether they are the winning bidder or not. In addition, the auctioneer will inform the winning bidder of the pre-auction of the amount to be paid.

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[1] The auction software uses the algorithms specified in the Appendix for determining the winning bidders and prices.

5. Principal stage of the main auction

5.1. Bids and submission of bids

5.1.1. During the principal stage of the main auction, bidders will submit combinatorial package bids for specific or abstract frequency blocks in the different categories. These bids are defined by ten parameters, specifically the bid amount and the number of frequency blocks in the categories A1 to C3 which the bidder wishes to acquire at that price (NB: The number of blocks may also be zero). A bid represents a binding offer on the part of the bidder to pay at most the specified price for a given combination of frequency blocks.

- In the primary bidding stage (clock stage), bidders will place package bids for a certain number of frequency blocks (which may also be zero) in each of the categories at a bid amount which is calculated based on the prices set by the auctioneer for the current round.
- In the sealed-bid stage (supplementary stage), bidders will submit combinatorial package bids for a number of frequency blocks in each of the categories; bidders will be allowed to choose freely the number of blocks as well as the bid amount during that stage, but within the limits defined by the activity rules (cf. Rule 5.9).

5.1.2. The auctioneer may begin the principal stage immediately with the sealed-bid stage. In such a case, the clock stage and the restrictions specified in Rule 5.9.6 will be eliminated entirely in accordance with Section 5.9.8. The auctioneer will only do so if convinced that it is in the general interest of an efficient frequency assignment procedure.

5.1.3. Bids will be submitted electronically using the auction software (cf. Section 7). The submission of bids by telephone will only be permitted in exceptional cases (e.g. technical difficulties). The auctioneer will be responsible for deciding whether such an exception is warranted.

5.1.4. During the clock stage, the bid amounts themselves cannot be chosen freely. Instead, in each round bidders will be required to nominate the desired number of frequency blocks (which may be zero) in each of the categories, and the bid amount will be calculated using the price set by the auctioneer for the current round (i.e. the number of blocks in a category is multiplied by the current round price for that category; the resulting amounts for each category are added up to yield the bid amount).

5.1.5. The current round price for a frequency block in a category will be set by the auctioneer (cf. Rule 5.7). The software will calculate the bid amount automatically when the package bid is submitted during the clock stage.

5.1.6. During the sealed-bid stage the bid amount can be chosen freely within the limits defined by the activity rules, and bids are to be submitted in accordance with Rule 5.9.

5.1.7. The amount of each bid submitted during the principal stage must comply with the rules regarding the bank guarantee as specified in Section 5.3.5 of the Tender Documentation. Bids which exceed the bidding limit arising from the respective bank guarantee will not be considered valid.

5.2. Bidding eligibility

5.2.1. The bidder's eligibility will determine the maximum number of frequency blocks on which that bidder can be active in any one round.

5.2.2. During the clock stage, a bidder will be allowed to be active on any combination of frequency blocks as long as the total eligibility points associated with the frequency blocks contained in that bidder's package bid do not exceed that bidder's current eligibility and Rule 5.5 is not violated.

5.2.3. Each bidder's eligibility in the first round of the clock stage will be determined on the basis of the information provided in the bidder's application (cf. Section 4.3 of the Tender Documentation). In subsequent rounds, the current eligibility of each bidder will be determined based on the activity rules.

5.2.4. During the sealed-bid stage, a bidder will be allowed to be active on any combination of frequency blocks as long as the total number of eligibility points associated with the package does not exceed that bidder's eligibility in the first round of the clock stage and the bid amounts adhere to the limits specified in the activity rules (cf. Rule 5.9).

5.3. Activity rules during the clock stage

5.3.1. The activity of a bidder in a given round is defined as the total number of eligibility points associated with the frequency blocks in the bidder's package bid submitted in that round (cf. Rule 3.1.1). The eligibility in a given round is equal to the bidder's activity in the previous round.

5.3.2. Bidders that do not submit a combinatorial package bid (i.e. that submit a "zero bid") will be excluded from further participation in the clock stage. However, such bidders will still be allowed to submit supplementary bids in the sealed-bid stage (cf. Rule 5.8.2), and the bids submitted by such bidders during the clock stage will still be taken into account in the winner determination procedure (cf. Rule 5.10).

5.4. Round extensions during the clock stage

5.4.1. At the beginning of the principal stage of the main auction, each bidder will be entitled to three round extensions.

5.4.2. These extensions provide a bidder with additional time to submit a bid during the clock stage. If a bidder that still has eligibility points as well as round extensions at their disposal does not submit a bid during the time allotted for a given round, then the software will automatically extend the round by a maximum of 30 minutes and subtract one of that bidder's round extensions. Round extensions will only be subtracted from those bidders that do not manage to submit a bid during the time allotted for a given round.

5.4.3. Bidders that do submit a bid during the time allotted for the round will not be able to revise their bids during the additional (extension) time. Bidders that have no more round extensions and do not manage to submit a bid during the time allotted for the round will not be allowed to submit a bid during an extension. In such cases, the auction software will automatically enter a zero bid for those bidders, and they will be excluded from further participation in the clock stage (cf. Rule 5.3.2).

5.4.4. Rounds in which one or more bidders make use of a round extension will come to an end no later than either 30 minutes after the originally scheduled end time, or once all bidders that used a round extension during that round have successfully entered a bid, whichever is shorter. The auction software will automatically enter a zero bid for

bidders that exercise their extension right and fail to submit a bid during the round extension period (see Rule 5.3.1).

5.4.5. No more than one round extension per bidder may be applied in each round.

5.5. Bidding restrictions

5.5.1. In addition to the restrictions deriving from the bidder's eligibility (see Rule 5.2), combinatorial bids will be subject to the restriction that one bid must not contain both block B1 and block B3.

5.5.2. Bidders must not submit (combinatorial) package bids that, including the spectrum won in the pre-auction, contain

- a spectrum exceeding 2x70 MHz, i.e. a package bid may contain a maximum of 14 frequency blocks (for a winner of the pre-auction the maximum is 12 frequency blocks) of the categories A1, A2, A3, B1, B2, B3, C1, C2 and C3;
- a spectrum exceeding 2x35 MHz below 1 GHz, i.e. a package bid may contain a maximum of seven frequency blocks (for a winner of the pre-auction the maximum is five frequency blocks) of the categories A1, A2, A3, B1, B2 and B3;
- a spectrum exceeding 2x20 MHz in the 800 MHz band, i.e. a package bid may contain a maximum of four frequency blocks (for a winner of the pre-auction the maximum is two frequency blocks) of the categories A1, A2 and A3.
- a spectrum exceeding 2x30 MHz in the 900 MHz band, i.e. a package bid may contain a maximum of six frequency blocks of the categories B1, B2 und B3.

5.6. Valid bids during the clock stage

5.6.1. A bid will be considered valid only if it is submitted within the time period defined by the auctioneer for the given round or during the extended time allotted to a bidder that exercises its round extension right (cf. Rule 5.4). This also applies to the submission of bids by telephone.

5.7. Round price determination during the clock stage

5.7.1. In the first round of the clock stage, the price of frequency blocks in each category will be set equal to the minimum bid.

5.7.2. In the following rounds of the clock phase, the prices will be increased for those categories where excess demand exists, i.e. where total demand (i.e. the total number of requested frequency blocks based on the package bids submitted) exceeds the number of frequency blocks available. If total demand is equal to or less than the number of available frequency blocks, the price for the category will remain unchanged.

5.7.3. Prices will not be reduced; this means that the price of either category in any given round of the clock stage cannot be lower than the price of that category in the previous round.

5.7.4. The amount by which the round price is increased will be determined by the auctioneer. The price increase will generally be between 2% and 15%, but the auctioneer reserves the right to apply different price increments. All prices will be set in whole euro amounts.

5.8. End of the clock stage

5.8.1. The clock stage ends:

- automatically if no excess demand exists in any of the categories; or
- by decision of the auctioneer, even if excess demand still exists in one or several categories.

The auctioneer will end the clock stage only if convinced that an immediate transition to the sealed-bid stage is in the general interest of an efficient frequency assignment procedure.

5.8.2. In both cases, the auction will proceed immediately to the sealed-bid stage (as described below), and no further rounds will be held in the clock stage.

5.9. Activity rules and valid bids for the sealed-bid stage

5.9.1. A bid will be considered valid only if it is submitted within the time period defined by the auctioneer for the sealed-bid stage. This also applies to the submission of bids by telephone.

5.9.2. The bids submitted in the sealed-bid stage must be in whole euro amounts.

5.9.3. Sealed supplementary bids can be submitted for:

- combinations of frequency blocks on which the bidder placed bids during the clock stage; and
- combinations of frequency blocks on which the bidder could have bid during the clock stage based on that bidder's eligibility (in the first round of the clock stage) and the rules set out in Rule 5.5.

Only one sealed package bid can be submitted for any particular combination of frequency blocks. The TKK has not yet decided whether the number of supplementary bids will be limited. However, at least 2,000 supplementary bids will be permitted.

5.9.4. The minimum amount of a sealed combinatorial package bid will be either the sum of the minimum bids (cf. Rule 3.2.2) for the frequency blocks included in the particular combination, or the highest clock bid submitted for that combination during the clock stage.

5.9.5. The following definitions are relevant for the determination of caps on bid amounts during the sealed-bid stage:

- "Zero bid": a bid of zero for a combination containing no frequency blocks.
- "Anchor round": the anchor round for a given combination C is the round in which the bidder last had sufficient eligibility to bid on that combination of frequency blocks.
- "Anchor combination": the anchor combination for a given combination C is the package on which the bidder placed a bid instead of combination C in the anchor round; the anchor combination is denoted as C'. C' can be a zero bid.
- "Anchor bid": the anchor bid for a given combination C is the highest bid received for the anchor combination (i.e. for C'). The anchor bid is therefore either the supplementary bid submitted for the anchor combination during the sealed-bid stage or the highest clock bid submitted for the anchor combination (if no supplementary bid is placed on this combination).
- "Last clock bid": the last non-zero bid submitted in the clock stage.

5.9.6. The bid amounts for sealed combinatorial package bids are subject to caps according to the provisions below and the definitions in Rule 5.9.5:

- Bidders that were active in the last round of the clock stage can increase the bid amount for the package they submitted in the last clock round as desired. If a bidder placed its last clock bid in an earlier round, that bidder can submit a sealed supplementary bid on the same combination of frequency blocks, but the bid will be capped at the value of that combination at the prices set in the round immediately following the round in which the bidder placed its last clock bid.
- The bid amount placed on any other combination of frequency blocks is capped relative to its respective anchor bid.
- The bid amount for combination C must not be higher than the corresponding anchor bid plus the price difference between combination C and the corresponding anchor combination C' at the prices in the anchor round.

This means that bidders that remain active until the end of the clock stage can increase the caps for sealed supplementary bids (indirectly) by submitting a higher bid on the combination they bid on in the last clock round. Bidders that remain active in the last round of the clock stage may increase their bid for that combination by any amount. For combinations requiring fewer eligibility points than (or the same number as) the combination bid on in the last clock round, the last round of the clock stage will be considered the anchor round, and the combination of the last clock bid will be considered the anchor combination.

Bidders that do not remain active until the last round of the clock stage can increase the last clock bid up to the amount they would have had to bid in the round when they first submitted a zero bid. All other sealed supplementary bids for combinations with more eligibility points than the combination of the last clock bid will be limited relative to their respective anchor combinations. For combinations that are associated with fewer eligibility points than (or the same number of eligibility points as) the combination of the last clock bid, the first round of the clock stage in which a zero bid was placed will be considered to be the anchor round, and the zero bid will be considered to be the anchor combination.

5.9.7. The auctioneer reserves the right to relax this restriction at the beginning of the sealed-bid stage for those supplementary bids where the anchor combination is not a zero bid. The limit will be relaxed by multiplying the value difference (between combination C and anchor combination C') added to the anchor bid in accordance with Rule 5.9.6 by a factor of (if the value difference is greater than zero) or $1/\alpha$ (if the value difference is less than zero). The auctioneer will do so only if convinced that it is in the general interest of an efficient frequency assignment procedure.

5.9.8. The auctioneer also reserves the right to suspend the limits specified in Rule 5.9.6. entirely, that is, to eliminate all upper limits on bid amounts for sealed supplementary bids.[2] The auctioneer will suspend the limits arising from Rule 5.9.6 entirely only if convinced that it is in the general interest of an efficient frequency assignment procedure.

[2] This does not affect the provisions on the bank guarantee (see Section 5.1.7).

The Appendix contains a number of examples that illustrate in detail the application of the activity rules.

5.10. Determination of winning bidders

5.10.1. After the end of the sealed-bid stage, the auctioneer will determine the combination of bids (among all bids submitted during the clock stage and the sealed-bid stage) which meets the following conditions:

- At most one combinatorial package bid per bidder is included.
- The total number of frequency blocks requested in each category must not be larger than the number of blocks available in that category.
- The total amount consisting of the sum of the bid amounts and the value of minimum bids for any unsold frequency blocks is not smaller than the total amount of any alternative combination of bids that meets the two aforementioned conditions.

5.10.2. If there is only one combination of combinatorial package bids that meets the conditions above, this will be considered the winning combination of bids.

5.10.3. If multiple combinations of combinatorial package bids meet the conditions set out in Rule 5.10.1, then the combination of (combinatorial) package bids that is associated with the largest number of eligibility points will be considered to be the winning combination of bids.

5.10.4. If multiple combinations of combinatorial package bids meet the conditions defined under Rule 5.10.1 and are associated with the same (largest) number of eligibility points, the combination of package bids that includes the largest number of bidders will be considered the winning combination of bids.

5.10.5. If multiple combinations of combinatorial package bids meet the conditions defined under Rule 5.10.1 and are assigned the same (largest) number of eligibility points and include the same (largest) number of winners, the winning combination of bids will be determined by a process of random selection.

5.10.6. The winning bidders will be those whose bids are part of the winning combination of package bids. The winning bidders will receive the number of frequency blocks in each category associated with their winning bids at the base price calculated according to Rule 5.11.

A description of the algorithm used to determine the winning bidders is provided in the Appendix.

5.11. Calculation of base prices

5.11.1. For each winning package bid, a base price will be calculated, which is the amount to be paid by the winning bidder for its winning bid. This means that base prices will not be calculated for individual abstract frequency blocks in each category. Base prices will also be calculated jointly for all winning bidders.

5.11.2. The base price for each winning bid must be at least as high as the total of the minimum bids for the combination of frequency blocks included in the winning bid. The base price for each winning bid cannot be higher than the amount of the corresponding winning bid.

5.11.3. Base prices are the lowest overall prices which:

- meet the condition set out in Rule 5.11.2; and

- ensure that the winning combination of bids at the respective base prices meets the conditions set out in Rule 5.10.1 where, in determining the base price of the winning bidder, the other bids of that bidder are reduced by the difference between the winning bid and the base price (or, in case this would result in a negative value, the other bids are set to zero).

This means that base prices are the lowest prices the winning bidders would have had to bid (jointly) in order for their respective bids to be successful.

5.11.4. If there is only one combination of base prices that meets the conditions under Rule 5.11.2 and Rule 5.11.3, then those base prices (rounded up to the next whole euro amount) will be set as the base prices.

5.11.5. If there are multiple combinations of base prices that meet those conditions, then the combination of base prices (rounded up to the next whole euro amount) that is closest to the opportunity costs of each winning bid (calculated individually) will be defined as the relevant combination of base prices. The opportunity costs of each winning bid are calculated individually and refer to the lowest bid that complies with Rule 5.11.2 and would ensure that the winning combination of bids meets the conditions of Rule 5.10.1 if all other winning bidders were to pay their original bid amount.

A description of the algorithm used to calculate base prices is provided in the Appendix.[3]

5.12. Information released to bidders

5.12.1. At the beginning of the first bidding round in the principal stage, each bidder will be informed of the number of blocks available in the A2 category (i.e. whether there was a winning bidder in the pre-auction). Additionally, each bidder will be informed of their bidding eligibility. This information will not be communicated to other bidders, nor will the auctioneer communicate the total number of eligibility points held by all bidders.

5.12.2. At the beginning of each round in the clock stage, the auctioneer will release the following minimum information to each bidder:

- the current round;
- the time at which the round ends;
- the price of a frequency block in each category in the current clock round;
- the number of eligibility points currently held by the bidder;
- the remaining number of round extensions held by the bidder.

5.12.3. After the end of each round in the clock stage, the auctioneer will release the following minimum information to each bidder:

- a summary of the combinatorial bid submitted;
- the number of eligibility points currently held by the bidder; and
- the remaining number of extensions held by the bidder.

[3] Due to numerical inaccuracies in the automated calculation of the base price, minimal deviations from the theoretical base price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated base price. See also the explanations on the calculation of base prices in the Appendix.

5.12.4. The auctioneer will inform each bidder accordingly once the clock stage has ended (cf. Rule 5.8).

5.12.5. At the beginning of the sealed-bid stage, the auctioneer will communicate at least the following information to each bidder:

- The time at which the sealed-bid stage ends;
- The degree to which bidding limits are to be relaxed (i.e. the value of parameter α in accordance with Rule 5.9.7).

5.12.6. If convinced that this is conducive to an efficient frequency assignment procedure, the auctioneer may at its discretion additionally release the following information to the bidders before the start of the sealed-bid stage:

- excess supply in the individual categories in the last clock round, if applicable;
- excess demand in each category for each round of the clock phase.

5.12.7. After the end of the principal stage (i.e. at the end of the sealed-bid stage), the auctioneer will release the following minimum information to each bidder:[4]

- the identity of each winning bidder of the principal stage and, if applicable, the winning bidder of the pre-auction; and
- the number of frequency blocks won in each category by the individual winning bidders in the principal stage and, if applicable, in the pre-auction.
- Each winning bidder of the principal stage will be informed of the base price for that bidder's winning bid. This information will not be communicated to other bidders.

6. Assignment stage

6.1. Assignment stage procedure

6.1.1. The purpose of the assignment stage is to determine how the available frequencies will be distributed amongst the winning bidders of the principal stage and/or the winning bidder of the pre-auction, and to determine the price the bidders will ultimately pay for the frequencies they have won.

6.1.2. The assignment stage consists of two rounds of bidding:

- In the first round of bidding, the frequency blocks in the C1, C2 and C3 categories will be assigned simultaneously for the period from 2013 to the end of 2019.
- In the second round of bidding, the frequency blocks in the A2/A3 and B2 categories (as far as required) as well as in the C1 to C3 categories will be assigned simultaneously for the period starting in 2020.

[4] Usually, the results should be available in less than one minute. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results or additional processing capacity may be required. In such an improbable case, the submission of bids will be concluded and the results will be announced at a later point in time.

6.2. Assignment of the frequency blocks in the C1, C2 and C3 categories for the period from 2013 to the end of 2019

6.2.1. If there is only one winning bidder in the C1 category, then no assignment bid will be necessary for that category. If the winning bidder wins only one frequency block, that bidder will automatically be assigned the frequency block LC02.

6.2.2. If more than one bidder acquires abstract frequency blocks in the C1 category in the principal stage, the auctioneer will provide each winning bidder with a complete list of relevant assignment bid options for specific frequency blocks. These options will meet the condition that the number of frequency blocks assigned must be equal to the number of abstract frequency blocks in the C1 category that the bidder won in the principal stage.

6.2.3. The auctioneer will provide each winning bidder of frequency blocks in the C2 category with a complete list of relevant assignment bid options for specific frequency blocks. These options will meet the condition that the number of frequency blocks assigned must be equal to the number of abstract frequency blocks in the C2 category that the bidder won in the principal stage.[5]

6.2.4. If the number of frequency blocks won in the C3 category in the principal stage is lower than the number of available frequency blocks (cf. Section 4.1 of the Tender Documentation), the frequency blocks will be assigned consecutively from frequency block LC11 upwards; this means that any spectrum not acquired in this category will be in contiguous blocks at the upper end of the frequency range.

6.2.5. If there is only one winning bid in the C3 category, then no assignment bid will be necessary for that category. The winning bidder will be assigned the relevant frequency blocks automatically in accordance with Rule 6.2.4.

6.2.6. If more than one bidder acquires abstract frequency blocks in the C3 category in the principal stage, the auctioneer will provide each winning bidder with a complete list of relevant assignment bid options for specific frequency blocks. These options must meet the following conditions:

- The number of frequency blocks assigned must be equal to the number of abstract frequency blocks in the C3 category that the bidder won in the principal stage.
- Assignment options must not prevent the assignment of contiguous frequency blocks to other bidders that in the principal stage won abstract frequency blocks in the category or prevent the retention of contiguous unassigned spectrum at the upper end of the frequency range.

6.3. Assignment of the frequency blocks in the A2/A3 and B2 categories as well as in the C1 to C3 categories for the period starting in 2020

6.3.1. The spectrum allocated in the pre-auction will be treated as two abstract frequency blocks in the A2 category. For the purpose of determining assignment options, the

[5] Usually, the assignment bid options will be available in less than one second. In certain situations (e.g. more than ten winning bidders), this process may take significantly longer, or additional processing capacity may be required. In such an improbable case, delays may arise in the auction procedure.

frequency blocks of A2 and A3 will be combined and assigned jointly (cf. Section 4.1 of the Tender Documentation).

6.3.2. For the A2/A3 and B2 categories, the frequency blocks able to be assigned directly on the basis of the following rules will first be identified:

- If a winning bidder of the specific block in the A1 category wins frequency blocks in the A2 or A3 category in the pre-auction and/or the principal stage, that bidder will be assigned the number of frequency blocks won in the A2 or A3 category consecutively, from the lower end of the A2 category upwards (that is, starting with LA02).
- If a winning bidder of the specific block in the B1 category wins frequency blocks in the B2 category, that bidder will be assigned the number of frequency blocks won in the B2 category consecutively, from the lower end of the B2 category upwards (that is, starting with LB02).
- If a winning bidder of the specific block in the B3 category wins frequency blocks in the B2 category, that bidder will be assigned the number of frequency blocks won in the B3 category consecutively, from the upper end of the B2 category upwards (that is, starting with LB06).

6.3.3. Bidders that are directly assigned frequency blocks in the A2/A3 and/or B2 category in accordance with Rule 6.3.2 will not participate any further in the assignment of frequency blocks in those categories.

6.3.4. If the number of frequency blocks in the A2/A3 and B2 categories exceeds the number of frequency blocks acquired by bidders in the principal stage, the frequency blocks will be assigned consecutively, from the first frequency block which has not been assigned directly in accordance with Rule 6.3.2 upwards. This means that any spectrum not acquired in these categories will be in contiguous blocks at the upper end of the respective frequency range that has not been directly assigned.

6.3.5. If there is only one winning bidder in the respective category that is not the winning bidder of A1 and/or B1 or B3, then no assignment bid will be necessary for that category. The winning bidder will be assigned the relevant frequency blocks automatically in accordance with Rule 6.3.4.

6.3.6. If more than one bidder that is not the winning bidder of A1 and/or B1 or B3 acquires in the principal stage abstract frequency blocks in those categories, the auctioneer will provide each winning bidder with a complete list of relevant assignment bid options for specific frequency blocks. These options must meet the following conditions:

- The number of frequency blocks assigned must be equal to the number of abstract frequency blocks in the A2/A3 and/or B2 category that the bidder won in the principal stage and in the pre-auction.
- Assignment options must not prevent the assignment of contiguous frequency blocks to other bidders that in the principal stage and the pre-auction won abstract frequency blocks in the category or prevent the retention of contiguous unassigned spectrum at the upper end of the frequency range in accordance with Rule 6.3.4.

6.3.7. For the purpose of assigning the frequency blocks in the C1 to C3 categories starting in 2020, the frequency blocks won in the principal stage in each category will be added up for each bidder. This means that the C1 to C3 categories will be combined into one single category C, in which all 15 frequency blocks of the 1.8 GHz band will be up for assignment. The number of frequency blocks won by a bidder in that

category will be determined as the sum of frequency blocks won by the bidder in the C1, C2 and C3 categories.

6.3.8. If any frequency blocks remain unsold in the C category, unsold frequency blocks will be assigned contiguously within the band.

6.3.9. The auctioneer will provide each winning bidder in that category with a complete list of relevant assignment bid options for specific frequency blocks. These options must meet the following conditions:

- The number of frequency blocks assigned must be equal to the number of abstract frequency blocks in the C category that the bidder won in the principal stage.
- Assignment options must not prevent the assignment of contiguous frequency blocks to other bidders that won abstract frequency blocks in the category in the principal stage.
- If any frequency blocks in those categories remain unsold, assignment options must not prevent the arrangement of these unsold frequency blocks contiguously within the band.

6.4. Bids and submission of bids

6.4.1. For each bidder, the auction software will determine all possible and relevant assignment options for the specific frequency blocks in each category; these options must meet the conditions defined under Rules 6.2 and 6.3. In the auction procedure, a bidder's assignment options will be unambiguously indicated in the list of the frequency blocks associated with the respective assignment option.

6.4.2. In the assignment stage, the winning bidders from the pre-auction or the principal stage will, where required, submit sealed bids for the assignment options defined for them in each category. If there is only one assignment option for a bidder in a given category, no bid will be necessary. The bidder will be automatically assigned the relevant frequency blocks. Bidders that have no or only one assignment option in a given category are not entitled to submit assignment bids for that category.

6.4.3. Bids will be submitted electronically using the auction software (cf. Section 7). The submission of bids by telephone will only be permitted in exceptional cases (e.g. technical difficulties). The auctioneer will be responsible for deciding whether such an exception is warranted.

6.4.4. Bidders may freely choose the bid amounts for packages during the assignment stage. If a bidder does not submit a bid on a possible assignment option defined for that bidder by the auction software, the auction software will automatically generate a zero bid for that assignment option.

6.5. Valid bids

6.5.1. During the assignment stage, a bid will be considered valid only if it is submitted within the time period defined by the auctioneer for the assignment bidding round. This also applies to the submission of bids by telephone.

6.5.2. In the assignment stage, bids must be submitted in whole euro amounts. The minimum bid in the assignment stage is EUR 0. Bids are not subject to an upper limit.

6.5.3. For each assignment option on which a bidder does not bid, the auction software will automatically generate a bid with an amount of EUR 0 (i.e. a "zero bid"). If a bidder does not submit any bid within the time period defined by the auctioneer, the auction

software will automatically place a zero bid for each assignment option in each category in which the bidder is allowed to bid.

6.6. Determination of winning bidders

6.6.1. After the end of the respective bidding round in the assignment stage, the auctioneer will determine, separately for all categories included in the bidding round, the combination of assignment bids (among all assignment bids submitted by the bidders and generated automatically by the auction software) which meets the following conditions in each category:

- In each category, exactly one assignment bid per bidder must be included.
- The assignment of frequency blocks in connection with the assignment bids must be mutually compatible and the frequency assignment must be unambiguous. This means that the assignment results in a bandplan where each bidder is assigned specific frequencies in the scope of the spectrum acquired in the principal stage in those categories, and frequencies are not assigned to more than one bidder.
- The assignment of unsold frequency blocks in connection with the assignment bids complies with the rules defined in Sections 6.2 and/or 6.3 on the assignment of unsold frequency blocks in that category.
- The sum of the assignment bid amounts must not be lower than the sum of any other alternative combination of assignment bids that meets the first two conditions.

6.6.2. If there is only one combination of assignment bids that meets the conditions above, that combination will be considered the winning combination of assignment bids.

6.6.3. If there are multiple combinations of assignment bids that meet the conditions under Section 6.6.1, a process of random selection will determine the winning combination of assignment bids.

6.6.4. Each bidder will be assigned the frequency blocks specified in the winning combination of assignment bids in their bid and will pay a top-up price determined in accordance with Rule 6.7.

6.7. Calculation of top-up prices

6.7.1. For each winning assignment bid, a top-up price will be calculated as the amount to be paid by each winning bidder in the principal stage for its winning assignment bid. Top-up prices will be calculated jointly for all bidders.

6.7.2. The top-up price for each winning assignment bid must be at least as high as the minimum bid for the respective assignment option (cf. Rule 6.5.2). The top-up price for each winning assignment bid cannot be higher than the amount of that bid.

6.7.3. Top-up prices are the lowest overall prices which:

- meet the condition set out in Rule 6.7.2; and
- ensure that the winning combination of bids meets the conditions specified in Rule 6.6.1 at the respective top-up prices where, for the determination of a winning bidder's top-up price, the other bids of that winning bidder are reduced by the difference between the respective winning bid and the top-up price (or, if this results in a negative amount, the other bids are set to zero).

This means that top-up prices are the lowest prices the winning bidders would have had to bid in order for their respective assignment bids to be successful.

6.7.4. If there is only one combination of prices that meets the conditions under Sections 6.7.2 and 6.7.3 , then those prices (rounded up to the next whole euro amount) will be set as the top-up prices.

6.7.5. If there are multiple combinations of top-up prices that meet those conditions, then the combination of top-up prices (rounded up to the next whole euro amount) that is closest to the opportunity costs of each winning assignment bid (calculated individually) will be selected as the relevant combination of top-up prices. The opportunity costs of each winning bid are calculated individually and refer to the lowest assignment bid that complies with Rule 6.7.2 and would ensure that the winning combination of assignment bids meets the conditions of Rule 6.6.1 if all other winning assignment bidders were to pay their original assignment bid amount.

A description of the algorithm used to determine top-up prices is provided in the Appendix.[6]

6.8. Overall price

6.8.1. The overall price to be paid by each winning bidder will be calculated as the sum of the base price (determined in the principal stage) and the top-up prices (determined in the assignment stage) as well as the price to be paid by a winning bidder in the pre-auction, if applicable.

6.9. Information released to bidders

6.9.1. At the beginning of each bidding round in the assignment stage, the auctioneer will release the following minimum information to each bidder eligible to participate:

- all options relevant for the bidder with regard to the assignment of specific frequency blocks;
- the time at which the assignment stage ends.

6.9.2. Once the first round of bidding in the assignment stage has ended, the auctioneer will inform each bidder of the frequency blocks which will be assigned to that bidder for the transition phase (i.e. the period from 2013 to the end of 2019).

6.9.3. At the end of the auction (i.e. at the end of the assignment stage), the auctioneer will release the following minimum information to each bidder:[7]

- the identity of each winning bidder;
- the frequency blocks to be assigned to each winning bidder in each category;
- the total price each winning bidder will have to pay for the frequency blocks, including an indication of the base price, the top-up price and, if applicable, the price from the pre-auction.

[6] Due to numerical inaccuracies in the automated calculation of the top-up price, minimal deviations from the theoretical top-up price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated top-up price. See also the explanations on the calculation of top-up prices in the Appendix.

[7] Cf. footnote no. 4.

7. Auction software

7.1. General information

Bids will be submitted electronically in decentralised manner using the auction software. The submission of bids by telephone will only be permitted in exceptional cases (e.g. technical difficulties). The auctioneer will be responsible for deciding whether such an exception is warranted. In such cases, the auctioneer will decide whether the current round is to be stopped and repeated, or whether the bidder experiencing technical difficulties should be allowed to submit bids without the auction software.

7.2. Procedure for auction rounds

From the bidders' perspective, each round will be divided into the following phases:

- Bid submission phase: Bids can only be placed during the bid submission phase. This phase will begin when the auctioneer announces the start of a round and when the bidding window is displayed. The bidding window will remain available until either the time allotted for the round (including any extension) has passed or the bidder has sent bids to the auction server.
- Waiting for round results: The successful submission of bids will then be confirmed. The bidder must then wait until the auctioneer either closes or discontinues the round.
- Display of round results: If the auctioneer closes the round, the bids will then be evaluated and the information defined in the auction rules will be displayed to the bidders.
- Discontinuation and repetition of a round: If the auctioneer discontinues the round (i.e. does not close the round normally), the bids submitted will not be evaluated (i.e. the bids are discarded); instead, the round will be repeated.
- Waiting for the start of the next round: once the results of a round have been evaluated, the bidder must wait until the next round begins. As soon as the auctioneer sets the scheduled start time for the next round, the start time will be displayed to the bidders. The scheduled start time is the earliest time at which the auctioneer can open the next round; the actual start of the next round will be triggered manually by the auctioneer and thus be slightly later than the scheduled time. The time between the rounds will be at least **ten minutes**.

In cases where a bidder has no connection to the server or encounters technical difficulties with the bidding client, bids will be submitted as specified in Section 7.7.

7.3. Submission of bids

The bids themselves will be submitted in two steps:

- In the first step, the bidder will enter one or more bids.
- In the second step, the bids will be transmitted to the auction server.

The specific form in which bids are to be entered will depend on the current stage in the auction procedure:

- Clock stage of the pre-auction: in this stage, participants must indicate whether they wish to acquire the frequency block at the price defined for the given round.

- Sealed-bid stage of the pre-auction: in this stage, the bidder can submit a supplementary bid for the frequency block.
- Clock stage of the main auction: in this stage, bidders must indicate how many abstract frequency blocks they would like to acquire in each category at the price defined for the given round.
- Sealed-bid stage of the main auction: in this stage, bidders may submit several supplementary package bids. In each package bid, the bidder must enter the bid amount and the number of frequency blocks desired in each category. In the sealed-bid stage, the bids can also be uploaded from a file.
- Assignment stage of the main auction: in this stage, the bidders may submit bids on assignment options for specific frequency blocks. The possible assignment options will be determined by the auction software.

Once the bidder has entered the bids, the software will require the bidder to confirm those bids.

Once a bid has been confirmed, it will be transmitted to the auction server and is thus considered binding. After that point, it will no longer be possible to modify, supplement or withdraw a bid.

7.4. Definitive time

The time on the auction server's clock will be considered definitive for the auction procedure. The auction server will be synchronised using the Network Time Protocol (NTP).

The time information displayed in the bidding window will be synchronised with the time on the server clock at regular intervals. During the bid submission phase, the end time for the round (according to the server's clock) as well as the time remaining in the round will be displayed on the bidding client. For technical reasons (e.g. latency between the client and server), this time may deviate slightly from the actual time remaining (according to the server's clock). For this reason, during the last minute of a given round the time remaining will be displayed only as "< 1 min".

Note: The system time usually displayed in the task bar on the bidder's computer will not be synchronised and may therefore differ significantly from the server time.

7.5. Duration of rounds

Details on the schedule and the duration of rounds will be provided in the binding Rules of Procedure.

7.6. Evaluation of bids and announcement of results

Once the bid submission phase for a round ends, the auctioneer can trigger the evaluation of bids in that round. Once the results of that evaluation are available, the information defined in the auction rules will be displayed to the bidders. Until they are evaluated, the bids remain unknown (also to the auctioneer).

Usually, the results should be available in less than one minute. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results, or additional processing capacity may be required. In such an improbable case, the submission of bids in the relevant round will be concluded and the results will be announced at a later point in time. This may lead to delays in the auction procedure.

7.7. Exception: submission of bids without the bidding software

The submission of bids without the bidding software will only be permitted in exceptional cases (e.g. technical difficulties). In such cases, the auctioneer must be informed by telephone immediately. The same time limits apply to the submission of bids with and without the bidding software. Details on the submission of bids by telephone will be provided in the binding Rules of Procedure.

7.8. Discontinuation of auction rounds

The auctioneer may discontinue an ongoing round at any time and repeat the round at a later point in time.

The bids placed in that round will then be considered void and remain unknown to all parties (including the auctioneer).

7.9. Interruptions of the auction

The auctioneer may interrupt the auction at any time and continue it at a later point in time. In such cases, the bidders will be informed of the reason for the interruption.

7.10. Messages to bidders

The auctioneer will be able to send messages to bidders through the auction software. Bidders will be alerted of unread messages by means of a symbol at the lower right of the bidding window.

8. Organisational framework

8.1. Auction

Organisational details regarding the auction will be provided in the binding Rules of Procedure.

8.2. Training sessions for bidders

8.2.1. Theory training session for bidders

The regulatory authority plans to hold theory training sessions for bidders on RTR's premises in July 2013. During that training session, the rules of the auction will be explained, and the bidders will be familiarised with the auction software. An individual appointment will be made for each bidder.

8.2.2. Practical training session for bidders

The regulatory authority plans to hold a practical training session for bidders in July 2013. The practical session will enable bidders to hold a test auction from their own premises.

It is recommended to use the same equipment (e.g. Internet access, phone, fax, screens/projectors, printers) in the practical training session as in the real auction. This way, technical problems can be identified and resolved in time.

8.3. Simulation tool

The regulatory authority will provide each (potential) applicant with access to a simulation tool for the purpose of verifying the procedures for determining winning bids and prices. For this purpose, applicants will be able to send an Excel file containing bids to a web-based interface. The simulation tool will then determine the winning bidders and the respective winning combinations and prices to be paid. Applicants will be provided with a description as well as a sample file for the simulation tool.

Once the tender submission period has ended, the regulatory authority will provide each bidder with free access to this simulation tool. As early as of May 2013, the regulatory authority will provide access to the simulation tool for a fee of EUR 3,000.

The simulation tool will not be available during the auction.

8.4. Additional bank guarantees during the auction

It will be possible to submit additional bank guarantees during the auction (cf. Section 5.3.5 of the Tender Documentation). Each bidder's eligibility will remain unaffected by any additional bank guarantees presented.

In the event that bank guarantees are presented only during the auction, such guarantees must be submitted by 12:00 noon local time (CET) on the business day preceding the bidding (Monday to Friday) in order to ensure sufficient time for the necessary verification procedures. Additional guarantees must be issued by the same bank which issued the guarantee enclosed with the application.

In this context, it should be noted that due to the design of the auction, the sealed-bid stage for supplementary bids may already take place on the second day of the auction. In such a case, it would be necessary to present additional bank guarantees by 12:00 noon on the first day of the auction.

Appendix A: Examples of maximum bids in the sealed-bid stage

The minimum bids used in the examples have been simplified and do not necessarily represent the minimum bids used in the auction.

Table 3: Lot structure and minimum bids for the following examples

Band	Category	Number	Eligibility points	Minimum bid
800 MHz	A1	1	2	20 million
	A2	4	2	20 million
	A3	1	2	20 million
900 MHz	B1	1	2	20 million
	B2	5	2	20 million
	B3	1	2	20 million
1800 MHz	C1	2	1	10 million
	C2	8	1	10 million
	C3	5	1	10 million

The following examples illustrate the application of the activity rules defined in Section 5.9 for the sealed-bid stage. It is assumed that no bidder has applied for the pre-auction and the frequency blocks of category A2 that are reserved for new entrants are to be allocated in the principal stage.

The restrictions for supplementary bids in the sealed-bid stage result from the activities in the clock stage. Those rounds are particularly important where bidders reduce their activity. The first example includes only one such round, whereas the second example is a more general one and includes several such rounds.

Example 1

For simplification purposes we assume that the prices of the different categories develop uniformly within a given band. This means that all frequency blocks within a given band have the same price in one round, regardless of their category within the range.

The bidder applies for eight eligibility points. In the first ten rounds of bidding, the bidder places bids on four frequency blocks in the A2 category in the 800 MHz band. In Round 11, the bidder places a bid on only three frequency blocks. The prices in Round 11 are as follows:

- EUR 30 million for a frequency block in the 800 MHz band (A1, A2 or A3 categories);
- EUR 40 million for a frequency block in the 900 MHz band (B1, B2 or B3 categories); and
- EUR 12 million for a frequency block in the 1800 MHz band (C1, C2 or C3 categories).

As a result of reducing the activity from eight to six eligibility points in Round 11, this round becomes the *anchor round* for all larger packages, that is, for packages with seven or eight eligibility points.

Until the end of the clock stage, the bidder submits bids on three A2 frequency blocks. The prices in the last round of the clock stage are as follows:

- EUR 35 million for a frequency block in the 800 MHz band;
- EUR 45 million for a frequency block in the 900 MHz band; and
- EUR 15 million for a frequency block in the 1800 MHz band.

The last package bid in the clock stage amounts to EUR 105 million for three frequency blocks in the A2 category. The bidder can choose to increase this bid by any amount, that is, to submit a sealed package bid for three frequency blocks in the A2 category at the amount of EUR 105 million + EUR X, with X being the amount by which the last clock bid is increased. All other package bids are limited relative to this *anchor bid*. The package of three frequency blocks in the A2 category (six eligibility points) is therefore the *anchor combination* for all supplementary bids.

- The bid amounts for all packages associated with seven or eight eligibility points are capped by the anchor bid (EUR 105 million + EUR X) and the prices in Round 11 (the *anchor round* for these packages).
- The bid amounts for all packages associated with six eligibility points or less are capped by the anchor bid (EUR 105 million + EUR X) and the prices in the last clock round (the *anchor round* for these packages).

Assuming that the bidder wishes to submit only bids on packages with at least two frequency blocks in the 800 MHz band and outside the 800 MHz band is only interested in frequencies in the C2 category, the bidder can submit the following sealed bids (with a maximum of eight eligibility points):

Table 4: Maximum bids in the sealed-bid stage – Example 1

A1	A2	A3	C2	Eligibility points	Maximum bid
1	1	0	0	4	Anchor bid – 35 million
0	1	1	0	4	Anchor bid – 35 million
0	2	0	0	4	Anchor bid – 35 million
1	1	0	1	5	Anchor bid – 35 million + 15 million
0	1	1	1	5	Anchor bid – 35 million + 15 million
0	2	0	1	5	Anchor bid – 35 million + 15 million
1	1	0	2	6	Anchor bid – 35 million + 30 million
0	1	1	2	6	Anchor bid – 35 million + 30 million
0	2	0	2	6	Anchor bid – 35 million + 30 million
0	**3**	**0**	**0**	**6**	**105 million (anchor bid)**
1	2	0	0	6	Anchor bid
0	2	1	0	6	Anchor bid
0	3	0	1	7	Anchor bid + 12 million
1	2	0	1	7	Anchor bid + 12 million
0	2	1	1	7	Anchor bid + 12 million
1	1	0	3	7	Anchor bid – 30 million + 36 million
0	1	1	3	7	Anchor bid – 30 million + 36 million
0	2	0	3	7	Anchor bid – 30 million + 36 million
0	3	0	2	8	Anchor bid + 24 million
1	2	0	2	8	Anchor bid + 24 million
0	2	1	2	8	Anchor bid + 24 million
1	1	0	4	8	Anchor bid – 30 million + 48 million
0	1	1	4	8	Anchor bid – 30 million + 48 million
0	2	0	4	8	Anchor bid – 30 million + 48 million

If the auctioneer relaxes the caps in accordance with Rule 5.9.7 and sets α = 2, the bidder may submit the following sealed bids:

Table 5: Maximum bids in the sealed-bid stage – Example 1 (α=2)

A1	A2	A3	C2	Eligibility points	Maximum bid
1	1	0	0	4	Anchor bid – 17.5 million
0	1	1	0	4	Anchor bid – 17.5 million
0	2	0	0	4	Anchor bid – 17.5 million
1	1	0	1	5	Anchor bid – 10 million
0	1	1	1	5	Anchor bid – 10 million
0	2	0	1	5	Anchor bid – 10 million
1	1	0	2	6	Anchor bid – 2.5 million
0	1	1	2	6	Anchor bid – 2.5 million
0	2	0	2	6	Anchor bid – 2.5 million
0	**3**	**0**	**0**	**6**	**105 million (anchor bid)**
1	2	0	0	6	Anchor bid
0	2	1	0	6	Anchor bid
0	3	0	1	7	Anchor bid + 24 million
1	2	0	1	7	Anchor bid + 24 million
0	2	1	1	7	Anchor bid + 24 million
1	1	0	3	7	Anchor bid + 12 million
0	1	1	3	7	Anchor bid + 12 million
0	2	0	3	7	Anchor bid + 12 million
0	3	0	2	8	Anchor bid + 48 million
1	2	0	2	8	Anchor bid + 48 million
0	2	1	2	8	Anchor bid + 48 million
1	1	0	4	8	Anchor bid + 36 million
0	1	1	4	8	Anchor bid + 36 million
0	2	0	4	8	Anchor bid + 36 million

Example 2

During the clock rounds the bidder exclusively submits bids on packages in the A2 category in the 800 MHz band. The bidder begins the auction with the eight eligibility points requested in the application and bids on four frequency blocks in the A2 category in the first round. The minimum bid per frequency block is EUR 20 million, and the package bid, therefore, comes to a total of EUR 80 million.

The bidder preliminarily continues to bid on four frequency blocks in the A2 category and reduces the bid to three frequency blocks in Round 5, at a current round price of EUR 24

million. The package of three frequency blocks in the A2 category is thus the bidder's *anchor combination* for all package bids requiring more than six eligibility points (that is, for all combinations requiring seven or eight eligibility points). The *anchor round* for those larger packages is Round 5.

In the following rounds, the bidder preliminarily continues to bid on three frequency blocks in the A2 category and reduces the bid to two frequency blocks in Round 10, at a current round price of EUR 29 million. The package of two frequency blocks in the A2 category is therefore the anchor combination for all packages requiring more than four but less than seven eligibility points (packages worth seven or more eligibility points are linked to the anchor bid for three frequency blocks in Round 5).

The bidder continues to bid on two frequency blocks in the A2 category until the end of the clock stage. In the last round of the clock stage, the bidder thus bids on two frequency blocks in the A2 category. The price of one such frequency block in the last round of the clock stage is EUR 35 million. The bidder's package bid for two frequency blocks is EUR 70 million. This package of two frequency blocks in the A2 category is the anchor combination for all packages requiring four eligibility points or less.

Table 6: Relevant round prices

Band	Category	Round 5	Round 10	Last clock round
800 MHz	A2	EUR 24 million	EUR 29 million	EUR 35 million
1800 MHz	C2	EUR 12 million	EUR 14 million	EUR 16 million

In the sealed-bid stage, the bidder can submit an unlimited bid for **two frequency blocks in the A2 category (four eligibility points),** as that was the bidder's last package bid during the clock stage and the bidder remained active until the end of that stage.

Let us assume that the bidder places a sealed bid of EUR 100 million for two frequency blocks in the A2 category. This will then be the anchor bid for the anchor combination of two frequency blocks in the A2 category. The anchor bid limits the maximum bid amounts for all supplementary bids with less than four eligibility points. The maximum bid amounts for the other packages on which the bidder wishes to bid are based on that anchor bid.

One frequency block in the A2 category (two eligibility points): The corresponding maximum bid is thus linked to the anchor bid for two frequency blocks and the value difference in the anchor round (in this case, the last clock round). The maximum bid for that package is calculated as follows: the value difference between the two packages of the last clock round is subtracted from the anchor bid (EUR 100 million). In this case, the value difference is the price of an individual block in the last round of the clock stage (EUR 35 million). This means that the maximum bid for a package with a frequency block in the A2 category is EUR 65 million.

Three frequency blocks in the A2 category (six eligibility points): The bidder was last eligible to bid on a package associated with six eligibility points in Round 10. Therefore, Round 10 is the anchor round for the supplementary bid for three frequency blocks in the A2 category. However, as the bidder placed a bid for two frequency blocks in Round 10 instead of continuing to bid on three frequency blocks, that bidder's eligibility was reduced to four eligibility points in Round 11. This means that the anchor bid for three frequency blocks is the highest bid for two frequency blocks in the A2 category, in this case the sealed supplementary bid of EUR 100 million. The maximum bid for three frequency blocks is

calculated as the anchor bid plus the value difference between the packages in the anchor round. The value difference is the price of an individual block in the A2 category in Round 10 (EUR 29 million). The maximum bid amount is the sum of the anchor bid (EUR 100 million) *plus* the value difference in the anchor round (EUR 29 million), that is, EUR 129 million. Let us assume that the bidder does not want to make full use of the maximum bid but rather submit a sealed bid of EUR 110 million for the package of three frequency blocks.

This bid is the new anchor bid for packages associated with more than six eligibility points. If the bidder had not submitted this sealed supplementary bid of EUR 110 million, the anchor bid for such packages would have been determined by the prices in Round 9.

Four frequency blocks in the A2 category (eight eligibility points): The bidder was last eligible to bid on a package associated with eight eligibility points in Round 5. Therefore, Round 5 is the anchor round. However, in that round, the bidder submitted a bid for three frequency blocks in the A2 category. The package bid for three frequency blocks is the anchor combination for packages associated with eight eligibility points (the maximum number of eligibility points available to the bidder). The bid on this anchor combination is, in turn, restricted by the anchor bid for three frequency blocks (in this case the sealed supplementary bid of EUR 110 million) and the value difference of the packages in Round 5, that is, the current round price for an individual frequency block (EUR 24 million). The maximum bid amount for four frequency blocks in the A2 category is therefore EUR 134 million. Let us assume that the bidder submits a sealed bid of EUR 130 million for the package of four frequency blocks in the A2 category.

The bidder may, of course, submit additional supplementary bids and bid on packages for which that bidder has not submitted bids in the clock stage. In this context, analogous restrictions apply to maximum bids.

Two frequency blocks in the C2 category (two eligibility points): Until the last round of the clock stage, the bidder was eligible to bid on such a package. The last clock round is therefore the relevant anchor round. The relevant anchor combination is the package of two frequency blocks in the A2 category, on which the bidder placed a bid in the last clock round and on which that bidder submitted a sealed supplementary bid of EUR 100 million. The maximum bid for the package of two frequency blocks in the C2 category is calculated as the anchor bid (EUR 100 million) plus the value difference between the package of two frequency blocks in the C2 category and the anchor combination (two frequency blocks in the A2 category) at the prices in the anchor round. Let us assume that the price for the C2 category was EUR 16 million in the last round of the clock stage. The value difference is calculated by subtracting the value of the anchor combination at the round price (2 x EUR 35 million = EUR 70 million) from the value of the combination desired by the bidder (2 x EUR 16 million = EUR 32 million). The maximum bid amount is therefore EUR 100 million + (EUR 32 million – EUR 70 million) = EUR 100 million – EUR 38 million = EUR 62 million.

One frequency block in the A2 category and three frequency blocks in the C2 category (five eligibility points): The bidder was last eligible to bid on a package associated with more than four eligibility points in Round 10. The corresponding anchor combination is the package of two frequency blocks in the A2 category; the associated anchor bid amounts to EUR 100 million. The value difference between the desired package and the anchor combination in the anchor round (Round 10) is (1 x EUR 29 million + 3 x EUR 14 million) - (2 x EUR 29 million) = EUR 13 million. This results in a maximum bid for the desired package of EUR 100 million + EUR 13 million = EUR 113 million.

Table 7: Maximum bids in the sealed-bid stage for packages of frequency blocks in the A2 and C2 categories

A2	C2	Eligibility points	Anchor round	Maximum bids (in million)
0	1	1	Last clock round	100 + (1 x 16 - 2 x 35) = 46
0	2	2	Last clock round	100 + (2 x 16 - 2 x 35) = 62
1	0	2	Last clock round	100 + (1 x 35 - 2 x 35) = 65
1	1	3	Last clock round	100 + (1 x 35 + 1 x 16 - 2 x 35) = 81
0	3	3	Last clock round	100 + (3 x 16 - 2 x 35) = 78
0	4	4	Last clock round	100 + (4 x 16 - 2 x 35) = 94
2	**0**	**4**	**Anchor bid in last clock round**	**Unrestricted** **Supplementary bid submitted: 100**
2	1	5	Round 10	100 + (2 x 29 + 1 x 14 - 2 x 29) = 114
1	3	5	Round 10	100 + (1 x 29 + 3 x 14 - 2 x 29) = 113
0	5	5	Round 10	100 + (5 x 14 - 2 x 29) = 112
0	6	6	Round 10	100 + (6 x 14 - 2 x 29) = 126
3	**0**	**6**	Round 10 – anchor bid for Round 5	**100 + (3 x 29 - 2 x 29) = 129** **Supplementary bid submitted: 110**
2	2	6	Round 10	100 + (2 x 29 + 2 x 14 - 2 x 29) = 128
1	4	6	Round 10	100 + (1 x 29 + 4 x 14 - 2 x 29) = 127
0	7	7	Round 5	110 + (7 x 12 - 3 x 24) = 122
3	1	7	Round 5	110 + (3 x 24 + 1 x 12 - 3 x 24) = 122
2	3	7	Round 5	110 + (2 x 24 + 3 x 12 - 3 x 24) = 122
1	5	7	Round 5	110 + (1 x 24 + 5 x 12 - 3 x 24) = 122
0	8	8	Round 5	110 + (8 x 12 - 3 x 24) = 134
4	0	8	Round 5	110 + (4 x 24 - 3 x 24) = 134
3	2	8	Round 5	110 + (3 x 24 + 2 x 12 - 3 x 24) = 134
2	4	8	Round 5	110 + (2 x 24 + 4 x 12 - 3 x 24) = 134
1	6	8	Round 5	110 + (1 x 24 + 6 x 12 - 3 x 24) = 134

Appendix B: Calculation of base prices

Description of the algorithm used to calculate base prices in accordance with Rule 5.11.

The procedure described below generates base prices which meet the conditions specified in Rule 5.11.

Let $n = 1,...,N$ be the bidders participating in the auction. The winner determination procedure in accordance with Rule 5.10 determines a set of winning bidders *W* and the total amount v consisting of the sum of all winning bids and the minimum bid value of any unsold frequency blocks. v^{-C} refers to the maximum total amount consisting of the sum of all bids and the total value of minimum bids for any unsold frequency blocks that would have been reached, if the bidders in set C had not participated in the auction. $\sigma(C) = (v - v^{-C})$ refers to the decrease in the total amount if the bidders in C had not participated in the auction.

a) For each winning bidder $j \in W$, calculate v^{-j} as the maximum total amount consisting of the sum of all bids and the total value of minimum bids for any unsold frequency blocks that would be reached if bidder j had not submitted a bid.

b) For each of the winning bidders $j \in W$ determine the maximum bid reduction (or "discount") $d_j^{\max} = \sigma(\{j\})$, where $\sigma(\{j\}) = \left(v - v^{-j}\right)$.

c) Generate a list of constraints B for a linear program and initialise it as $B = \left\{ \forall j \in W : d_j \le d_j^{\max}, \sum_{j \in W} d_j \le \sigma(W) \right\}$.

d) Solve the following maximisation problem: $\max \sum_{j \in W} d_j$ subject to the constraints in B and the constraint that the individual values of d_j cannot be negative. In general, many solutions to this maximisation problem may exist. If this is the case, pick a random maximum solution d^*.

e) Reduce the amounts of any bids submitted by bidder j by d_j^*. If this yields a negative bid amount, set this bid amount to zero.

f) Repeat the winner determination procedure in accordance with Rule 5.10 using the modified bid amounts and calculate the total amount resulting from the sum of such modified bid amounts and the minimum bid value of any unsold frequency blocks.

g) If the total amount resulting from the sum of originally winning bids using the modified bid amounts and the minimum bid value of any unsold frequency blocks does not yield a lower value than the total amount determined under Step f), the maximum bid reduction that may be granted to all of the winning bidders equals $D^* = \sum_{j \in W} d_j^*$

Proceed to Step i).

h) If the total amount resulting from the sum of originally winning bids using the modified bid amounts and the minimum bid value of any unsold frequency blocks yields a lower value than the total amount determined under Step k), then add the following constraint to the list of constraints *B*:

$$\sum_{j \in L} d_j \le \sigma(L)$$

where L denotes the set of bidders that are no longer among the winning bidders using the modified bid amounts. Proceed to Step d).

i) Solve the following minimisation problem: $\min \sum_{j \in W} \left(d_j - \sigma(\{j\})\right)^2$ subject to the constraints in B, the constraint that the individual values of d_j cannot be negative, and the constraint that the sum of the individual bid reductions must be equal to the maximum permissible overall sum of bid reductions, i.e. $\sum_{j \in W} d_j = D^*$. Let d^{**} be the solution to this minimisation problem.

j) Reduce the amounts of any bids submitted by bidder j by d_j^{**}. If this yields a negative bid amount, set this bid amount to zero.

k) Repeat the winner determination procedure in accordance with Rule 5.10 using the modified bid amounts and calculate the total amount resulting from the sum of such modified bid amounts and the minimum bid value of any unsold frequency blocks.

l) If the total amount resulting from the sum of originally winning bids using the modified bid amounts and the minimum bid value of any unsold frequency blocks does not yield a lower value than the total amount determined under Step k), then the relevant bid reductions will be d_j^{**} and the base prices will equal the bids reduced by those bid reductions, i. e. $p_j = b_j - d_j^{**}$ for each $j \in W$.

m) If the total amount resulting from the sum of originally winning bids using the modified bid amounts and the minimum bid value of any unsold frequency blocks yields a lower value than the total amount determined under Step k), then add the following constraint to the list of constraints B:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders that are no longer among the winning bidders using the modified bid amounts. Proceed to Step i).

Due to numerical inaccuracies in the automated calculation of base prices (minimum revenue core prices), minimal deviations from the theoretical base price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated base price. In very rare cases, the base price calculated may be a few euros more than the theoretical base price.

Usually, the results should be available in less than one minute. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results, or additional processing capacity may be required. In such an improbable case, the submission of bids will be concluded and the results will be announced at a later point in time.

Example 3: Calculation of base prices

In this example, six bidders (Alan, Ben, Carl, Doris, Emma and Fred) take part in the bidding process. None of the bidders submitted an application in the pre-auction, which means the frequency blocks reserved for a new entrant in the A2 category will now be allocated in the principal stage. During the principal stage, the bids listed in Table 8 are submitted. The minimum bids are the same as in the examples above (cf. Table 3).

Table 8: Combinatorial package bids in the principal stage – Example 3

Bidder	Package									Package bid (principal stage)
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	1	1	0	0	0	2	EUR 320 million
	1	1	0	1	1	0	0	0	2	EUR 250 million
Ben	0	2	0	0	2	1	1	4	0	EUR 320 million
	0	2	0	0	2	1	1	3	0	EUR 295 million
	0	1	0	0	2	1	0	4	0	EUR 185 million
	0	1	0	0	2	1	0	3	0	EUR 170 million
Carl	0	1	1	0	0	0	1	0	1	EUR 160 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 310 million
	0	0	0	0	2	0	0	4	2	EUR 300 million

The package bids highlighted in blue are the winning bids since they represent the (unique) combination maximising the total auction revenues that can be accommodated within the available spectrum, whereas any unsold frequency blocks are valued at their minimum bid. The total amount resulting from this combination is EUR 1.03 billion, including the sum of the winning bids (EUR 1.03 billion) and the total of the minimum bids for the unsold frequency blocks (EUR 0). The bidders that were not successful and their respective bids are highlighted in grey.

The description below follows the algorithm discussed in this Appendix.

Steps a) and b): for each winning bidder, calculate the total amount which would result from the winner determination procedure if that bidder had not submitted a bid. Determine the maximum bid reduction.

Thus, the first step towards finding the base prices is to calculate the maximum bid reductions for each of the winning bidders.

If Alan had not participated in the auction, the frequency blocks acquired by him would not have been allocated. Thus, the opportunity costs resulting from Alan's winning bid amount to EUR 100 million and the base price for Alan's winning bid is at least EUR 100 million.

Ben has acquired two frequency blocks in the A2 and B2 categories, as well as one frequency block in B3 and C1 and four frequency blocks in C2. If Ben had not participated in the auction, Alan would have acquired an additional frequency block in the A2 category and Doris could have acquired the two frequency blocks in the B2 category and the four frequency blocks in C2. As a result, there would have been another winning bidder (Doris) in addition to the existing winning bidders (Alan, Carl and Fred). In this case one frequency block in A2, one frequency block in B3 and one frequency block in the C1 category remain unsold. The sum of the winning bids in this second-best allocation is EUR 890 million and the total of the minimum bids for the unsold frequency blocks amounts to EUR 50 million. The total amount resulting from this second-best allocation is therefore EUR 940 million. This means that the maximum bid reduction is EUR 90 million (EUR 1.03 billion – EUR 940 million), and the opportunity cost of Ben's bid amounts to EUR 230 million.

If Carl had not taken part in the auction, the second-best result in terms of revenue maximisation would have been to assign another frequency block in the A2 category to Alan (with a package bid of EUR 320 million). In this second-best allocation one frequency block in A3, one frequency block in C1 and one frequency block in C3 remain unsold. The sum of the winning bids in this second-best allocation is EUR 940 million and the total of the minimum bids for the unsold frequency blocks amounts to EUR 40 million. The total amount resulting from this second-best allocation is therefore EUR 980 million. Thus, a maximum bid reduction of EUR 50 million is granted to Carl. Subtracting this value from his bid yields the opportunity cost of his winning bid, which equals EUR 110 million.

Fred has won two frequency blocks in B2, four frequency blocks in C2 and two frequency blocks in C3. If Fred had not participated in the auction, then Emma would have been able to acquire this package. This second-best allocation does not yield any unsold frequency blocks. Therefore, the total amount, if Fred does not take part in the auction, will be EUR 870 million, which yields a maximum bid reduction of EUR 160 million for Fred; the opportunity cost of Fred's winning bid is hence EUR 140 million.

The results of the calculation of the maximum bid reductions (Step b)) and the resulting potential base prices are shown in Table 9.

Table 9: Potential base prices – Example 3

Bidder	Next best maximum total amount	Maximum discount	Potential base price
Alan	EUR 880 m	EUR 1.03 bn - EUR 880 m = EUR 150 m	EUR 100 m
Ben	EUR 940 m	EUR 1.03 bn - EUR 940 m = EUR 90 m	EUR 230 m
Carl	EUR 980 m	EUR 1.03 bn - EUR 980 m = EUR 50 m	EUR 110 m
Fred	EUR 870 m	EUR 1.03 bn - EUR 870 m = EUR 160 m	EUR 140 m

Step c): we then generate a list of constraints and initialise it with the conditions that the individual discounts must not be higher than the maximum discounts calculated in the previous step, and that the overall sum of discounts over all bidders together must not be larger than the reduction in the total bid amount if the bidders had not taken part in the auction.

If none of the winning bidders had participated in the auction, Emma and Doris would have won and the total amount would have been EUR 480 million. This means that the list of constraints includes the following conditions:

$$d_{Alan} \geq 0$$
$$d_{Ben} \geq 0$$
$$d_{Carl} \geq 0$$
$$d_{Fred} \geq 0$$
$$d_{Alan} \leq 150Mio$$
$$d_{Ben} \leq 90Mio$$
$$d_{Carl} \leq 50Mio$$
$$d_{Fred} \leq 160Mio$$
$$d_{\{Alan,Ben,Carl,Fred\}} \leq 550Mio$$

Step d): maximise the sum of the individual discounts subject to these constraints.

The solution to the maximisation problem subject to the constraints is as follows:

$$d^{\star}_{Alan} = 150Mio$$
$$d^{\star}_{Ben} = 90Mio$$
$$d^{\star}_{Carl} = 50Mio$$
$$d^{\star}_{Fred} = 160Mio$$

Steps e), f) and g): reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero. Then determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and calculate the resulting total amount. If the original winning bids using the reduced bid amounts and the total of the minimum bids for the originally unsold frequency blocks do not yield a lower value, proceed to Step i).

If we reduce all of the winning bidders' package bids by the calculated maximum bid reductions and then calculate the resulting maximum total bid amount on the basis of those modified bid amounts, it becomes clear that the latter amount is no higher than the sum of the original winning bids less the corresponding bid reductions. The combination of the original winning bids therefore remains a potential winning combination (although it may be on par with other combinations). This means that the potential base prices shown in Table 9 represent the actual base prices, as shown in Table 10 below.

Table 10: Modified package bids – Example 3

Bidder	**Package**									**Modified bid (after application of discount)**
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	1	1	0	0	0	2	EUR 170 million
	1	1	0	1	1	0	0	0	2	EUR 100 million
Ben	0	2	0	0	2	1	1	4	0	EUR 230 million
	0	2	0	0	2	1	1	3	0	EUR 205 million
	0	1	0	0	2	1	0	4	0	EUR 95 million
	0	1	0	0	2	1	0	3	0	EUR 80 million
Carl	0	1	1	0	0	0	1	0	1	EUR 110 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 150 million
	0	0	0	0	2	0	0	4	2	EUR 140 million

Steps i), j), k) and l): minimise the sum of squared differences between the individual discounts and the individual maximum discounts $\sigma(\{j\}) = \left(v - v^{-j}\right)$, subject to the constraints and the condition that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts. Reduce the bid amounts for all of the winners' bids by the bid reduction calculated in this way; if this yields a negative bid amount, set this bid amount to zero. Determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and calculate the resulting total amount. If the original winning bids using the reduced bid amounts and the total of the minimum bids for the originally unsold frequency blocks do not yield a lower value than the total bid amount calculated in Step k), then the bid reductions found are the relevant bid reductions and the base prices are equal to the winning bids reduced by those bid reductions.

The quadratic optimisation yields the discounts found at the end of the linear optimisation. Those bid reductions do not change the result of the winner determination procedure and are

thus the discounts with which the corresponding base prices are calculated. The base prices are shown in Table 11.

Table 11: Base prices for winning bids – Example 3

Bidder	Base price
Alan	EUR 100 million
Ben	EUR 230 million
Carl	EUR 110 million
Fred	EUR 140 million
Total:	EUR 580 million

Example 4: Calculation of base prices – Minimisation of total value

Base prices are not generally identical to opportunity costs, as is the case in Example 3; in principle, it cannot be ensured that reducing the winning bids by the maximum discounts will leave the winning bids unchanged. Under these circumstances, the base prices need to be higher than the individual opportunity costs of winning bids.

Let us assume that an additional bidder, Greg, takes part in the auction, and hence a total of seven bidders (Alan, Ben, Carl, Doris, Emma, Fred and Greg) bid on frequency blocks. None of the bidders has submitted an application in the pre-auction, which means the frequency blocks reserved for a new entrant in the A2 category will now be allocated in the principal stage. During the principal stage, the bids listed in Table 12 are submitted.

Alan, Ben, Carl, Doris, Emma and Fred submit the same bids as in the previous example. Greg submits a combinatorial package bid for one frequency block in A2, four frequency blocks in C2 and three frequency blocks in C3, as well as another combinatorial package bid for one frequency block each in A1, A2 and C3. The winning combination of bids is again highlighted in blue.

Table 12: Combinatorial package bids in the principal stage – Example 4

Bidder	Package									Package bid (principal stage)
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	1	1	0	0	0	2	EUR 320 million
	1	1	0	1	1	0	0	0	2	EUR 250 million
Ben	0	2	0	0	2	1	1	4	0	EUR 320 million
	0	2	0	0	2	1	1	3	0	EUR 295 million
	0	1	0	0	2	1	0	4	0	EUR 185 million
	0	1	0	0	2	1	0	3	0	EUR 170 million
Carl	0	1	1	0	0	0	1	0	1	EUR 160 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 310 million
	0	0	0	0	2	0	0	4	2	EUR 300 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

The same bidders (Alan, Ben, Carl and Fred) win the same number of frequency blocks in the different categories as in the previous example. However, Greg's participation in the auction increases the base prices for the winning bids.

Steps a) and b): for each winning bidder, calculate the total amount which would result from the winner determination procedure if that bidder had not submitted a bid. Determine the maximum bid reduction.

If Alan had not participated in the auction, the combination of awarding one frequency block in each of the categories A1, A2 and C3 to Greg's bid and the other winning bids to the remaining bidders would be the combination of bids which maximises the revenue. In this allocation one frequency block in each of the categories B1, B2 and C3 remains unsold. Thus the sum of all winning bids would be equal to EUR 880 million and the sum of the unsold frequency blocks would amount to EUR 50 million. Therefore, the total amount resulting from this alternative allocation is EUR 930 million and the maximum bid reduction for Alan equals EUR 100 million. The potential base price of Alan's winning bid is EUR 150 million. Applying the same procedure to the other bidders results in the potential base prices for each winning bid as shown in Table 13.

Table 13: Potential base prices – Example 4

Bidder	Next best maximum total amount	Maximum discount	Potential base price
Alan	EUR 930 m	EUR 1.03 bn – EUR 930 m = 100 m	EUR 150 m
Ben	EUR 940 m	EUR 1.03 bn - EUR 940 m = 90 m	EUR 230 m
Carl	EUR 980 m	EUR 1.03 bn - EUR 980 m = 50 m	EUR 110 m
Fred	EUR 910 m	EUR 1.03 bn - EUR 910 m = 120 m	EUR 180 m

Step c): we then generate a list of constraints and initialise it with the conditions that the individual discounts must not be higher than the maximum discounts calculated in the previous step, and that the overall sum of discounts over all bidders together must not be larger than the reduction in the total bid amount if the bidders had not taken part in the auction.

If the winning bidders do not participate in the auction, Emma and Greg will win. In this case one frequency block in each of the categories A1, A3, B1 and B3 as well as three frequency blocks in A2 and B and two frequency blocks in C1 remain unsold. The sum of the winning bids in this alternative allocation is EUR 410 million and the total of the minimum bids for the unsold frequency blocks equals EUR 220 million. Therefore, the total amount resulting from this alternative allocation is EUR 630 million and the maximum bid reduction for all winning bidders taken together equals EUR 400 million.

The constraints on this linear program are therefore as follows:

$$d_{Alan} \geq 0$$
$$d_{Ben} \geq 0$$
$$d_{Carl} \geq 0$$
$$d_{Fred} \geq 0$$
$$d_{Alan} \leq 100Mio$$
$$d_{Ben} \leq 90Mio$$
$$d_{Carl} \leq 50Mio$$
$$d_{Fred} \leq 120Mio$$
$$d_{\{Alan,Ben,Carl,Fred\}} \leq 400Mio$$

Step d): maximise the sum of the individual discounts subject to these constraints.

The solution to this maximisation problem subject to the constraints is as follows:

$$d^{\star}_{Alan} = 100Mio$$
$$d^{\star}_{Ben} = 90Mio$$
$$d^{\star}_{Carl} = 50Mio$$
$$d^{\star}_{Fred} = 120Mio$$

Step e): reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero.

Subtracting the bid reductions from all of the winners' bids yields the following modified package bids:

Table 14: Modified package bids, first iteration – Example 4

Bidder	**Package**									**Modified bid (after application of discount)**
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	1	1	0	0	0	2	EUR 220 million
	1	1	0	1	1	0	0	0	2	EUR 150 million
Ben	0	2	0	0	2	1	1	4	0	EUR 230 million
	0	2	0	0	2	1	1	3	0	EUR 205 million
	0	1	0	0	2	1	0	4	0	EUR 95 million
	0	1	0	0	2	1	0	3	0	EUR 80 million
Carl	0	1	1	0	0	0	1	0	1	EUR 110 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 190 million
	0	0	0	0	2	0	0	4	2	EUR 180 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

Steps f) and g): then determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and the resulting total amount, and compare these with the original winning bids using the reduced bid amounts.

Again, we arrive at the combination (of the now modified bids) yielding the highest total amount. This combination is highlighted in blue in Table 14. In this combination one frequency block in each of the categories A3 and C1 as well as two frequency blocks in B2 remain unsold. In this combination the sum of the modified bids amounts to EUR 650 million and the sum of the unsold frequency blocks to EUR 70 million. Therefore, the modified total

amount resulting from this combination equals EUR 720 million. The sum of the modified original winning bids amounts to EUR 670 million and the total of the minimum bids for the originally unsold frequency blocks to EUR 0. The modified total amount of the original winning combination of bidders therefore differs from the highest total amount achievable with the modified bids. Compared to the original result, Carl and Fred are eliminated from the modified winning combination of bidders.

Step h): if there are winning bidders from the original winner determination procedure that are no longer among the winning bidders with the modified bids, then expand the list of constraints to include the condition that the sum of bid reductions for those bidders that are eliminated from the winning combination must not be higher than the reduction of the total bid amount if those bidders had not taken part in the auction. Proceed to Step d).

We therefore have to find the next best total amount on the basis of the original bids submitted (see Table 12) including the minimum bids for any unsold frequency blocks which would have arisen if Carl and Fred had not participated in the auction. This would yield EUR 910 million (in this case Greg would receive his first package beside the other original winning bidders, and one frequency block in each of the categories A3 and C1 as well as two frequency blocks in B2 would remain unsold). Therefore, $\sigma\left(Carl, Fred\right) = 1,03Mrd - 910Mio = 120Mio$. Accordingly, the maximum sum of bid reductions for Carl and Fred can be at most EUR 120 million. We therefore add the following constraint to our list:

$$d_{Carl} + d_{Fred} \leq 120Mio$$

Step d): maximise the sum of the individual discounts subject to these constraints.

One possible solution to the maximisation problem given the expanded list of constraints is as follows:

$$d^{\star}_{Alan} = 100Mio$$
$$d^{\star}_{Ben} = 90Mio$$
$$d^{\star}_{Carl} = 0Mio$$
$$d^{\star}_{Fred} = 120Mio$$

This is only one possible solution, as any combination of non-negative bid reductions (also referred to as discounts) for Carl and Fred that meet the condition $d_{Carl} + d_{Fred} \leq 120Mio$ (and, of course, the other constraints) maximises the sum of discounts.

Step e): reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero.

Subtracting these discounts from the winners' bids in Table 12 yields the modified winning bids indicated in the table below.

Table 15: Modified package bids, second iteration – Example 4

Bidder	Package									Modified bid (after application of discount)
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	1	1	0	0	0	2	EUR 220 million
	1	1	0	1	1	0	0	0	2	EUR 150 million
Ben	0	2	0	0	2	1	1	4	0	EUR 230 million
	0	2	0	0	2	1	1	3	0	EUR 205 million
	0	1	0	0	2	1	0	4	0	EUR 95 million
	0	1	0	0	2	1	0	3	0	EUR 80 million
Carl	0	1	1	0	0	0	1	0	1	EUR 160 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 190 million
	0	0	0	0	2	0	0	4	2	EUR 180 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

Steps f) and g): then determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and the resulting total amount, and compare these with the original winning bids using the reduced bid amounts.

Again we arrive at the combination (of the now modified bids) yielding the highest modified total amount. This combination is highlighted in blue in Table 15. In this combination one frequency block in each of the categories A1, B1 and C3, as well as three frequency blocks in B2 remain unsold. In this combination the sum of the modified bids amounts to EUR 660 and the sum of the unsold frequency blocks to EUR 110 million. Therefore, the modified total amount resulting from this combination equals EUR 770 million. The sum of the modified original winning bids amounts to EUR 720 million and the total of the minimum bids for the originally unsold frequency blocks to EUR 0. The total amount resulting from the modified original winning combination differs therefore from the highest total amount achievable with the modified bids.

Step h): if there are winning bidders from the original winner determination procedure that are no longer among the winning bidders with the modified bids, then expand the list of constraints to include the condition that the sum of bid reductions for those bidders that are eliminated from the winning combination must not be higher than the reduction of the total bid amount if those bidders had not taken part in the auction. Proceed to Step d).

Now Alan and Fred are eliminated from the original winning combination. On the basis of Table 12 we therefore have to identify the next best total amount if Alan and Fred had not participated in the auction. This would yield EUR 860 million (Greg would also receive his first package, as well as the other original winning bidders, and one frequency block in each of the categories A1, B1 and C3 as well as three frequency blocks in B2 would remain unsold). Therefore, $\sigma\left(Alan, Fred\right) = 1,03Mrd - 860Mio = 170Mio$; the maximum bid reduction (discount) which can be granted to Alan and Fred together is EUR 170 million. We now add the following constraint to our list:

$$d_{Alan} + d_{Fred} \leq 170Mio$$

Step d): maximise the sum of the individual discounts subject to these constraints.

A solution to the maximisation problem given the expanded list of constraints is as follows:

$$d^{\star}_{Alan} = 100Mio$$
$$d^{\star}_{Ben} = 90Mio$$
$$d^{\star}_{Carl} = 50Mio$$
$$d^{\star}_{Fred} = 70Mio$$

Steps d), e), and f): reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero. then determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and the resulting total amount, and compare these with the original winning bids using the reduced bid amounts.

Subtracting these discounts from the winners' bids in Table 12 yields the modified winning bids indicated in the table below.

Table 16: Modified package bids, third iteration – Example 4

Bidder	**Package**									**Modified bid (after application of discount)**
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	1	1	0	0	0	2	EUR 220 million
	1	1	0	1	1	0	0	0	2	EUR 150 million
Ben	0	2	0	0	2	1	1	4	0	EUR 230 million
	0	2	0	0	2	1	1	3	0	EUR 205 million
	0	1	0	0	2	1	0	4	0	EUR 95 million
	0	1	0	0	2	1	0	3	0	EUR 80 million
Carl	0	1	1	0	0	0	1	0	1	EUR 110 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 240 million
	0	0	0	0	2	0	0	4	2	EUR 230 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

The winning combination now remains the same, that is, Step f) does not yield a higher modified total amount than the original winning combination. The next step is quadratic optimisation.

Steps i), j), k) and l): minimise the sum of squared differences between the individual discounts and the individual maximum discounts $\sigma(\{j\}) = (v - v^{-j})$, subject to the constraints and the condition that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts. Reduce the bid amounts for all of the winners' bids by their corresponding discounts; if this yields a negative bid amount, set this bid amount to zero. Determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and calculate the resulting total amount. If the original winning bids using the reduced bid amounts do not yield a lower value than the total amount calculated in Step k), then the discounts found are the relevant discounts and the base prices are equal to the winning bids reduced by those amounts.

The quadratic optimisation yields the discounts found at the end of the linear optimisation. Those discounts do not change the result of the winner determination procedure and are

thus the discounts with which the corresponding base prices can be calculated. The base prices are listed in Table 17.

Table 17: Base prices for winning bids – Example 4

Bidder	Base price
Alan	EUR 150 million
Ben	EUR 230 million
Carl	EUR 110 million
Fred	EUR 230 million
Total:	EUR 720 million

The intuitive explanation for these base prices is as follows:

If, as in the previous example, the winning bidders were granted their respective maximum individual discounts, then Greg's bid, and the total of the minimum bids for the frequency blocks unsold at the time, would be higher than the sum of Fred's and Alan's or Fred's and Carl's modified winning bids, and thus it would maximise revenues if Greg's bid were accepted instead of Fred's and Alan's. This gives rise to additional constraints on the base prices:

- Alan's and Fred's base prices taken together must be higher than Greg's bid for this package (EUR 270 million) and the total of the minimum bids for the frequency blocks unsold at the time (EUR 110 million);
- Carl's and Fred's base prices taken together must be higher than Greg's bid for this package (EUR 270 million) and the total of the minimum bids for the frequency blocks unsold at the time (EUR 70 million).

This means that the total of base prices for Alan and Fred and for Carl and Fred must in each case be EUR 50 million higher than the sum of the individual opportunity costs for those bidders.

As the sum of all base prices is to be minimised (and thus the sum of all discounts maximised), there is only one solution in this example: Fred must pay the additional required amount of EUR 50 million on his own. Otherwise, both Alan and Carl would pay a higher base price, and the sum of base prices would thus be higher than if Fred alone paid the additional amount. If, for example, Fred only paid half of the additional EUR 50 million, then both Alan and Carl would each have to pay an additional EUR 25 million, and the sum of base prices would be EUR 745 million. This is higher than the sum of base prices in the scenario where Fred alone pays the EUR 50 million (EUR 720 million).

Example 5: Calculation of base prices – minimisation of distance from individual opportunity costs

Example 4 illustrated a case in which a group of winning bidders must pay an amount which is larger than the sum of individual opportunity costs in order to ensure that the combination of bids which maximises the total revenues remains unchanged.

In the previous example, the rule that the sum of discounts should be maximised (or, conversely, that the sum of base prices for all winning bids should be minimised) was sufficient in order to identify a unique combination of base prices. However, this is not always the case.

In the following example Carl no longer participates in the auction and Alan's first bid is modified to now include the B3 frequency block instead of the B1 frequency block. The corresponding bids are summarised in Table 18. None of the bidders has submitted an application in the pre-auction, which means the frequency blocks reserved for a new entrant in the A2 category will now be allocated in the principal stage. The winning combination of bids is again highlighted in blue. In this allocation one frequency block in each of the categories A2, A3, C1 and C3 remains unsold. Thus the sum of all winning bids is EUR 870 million and the total of the minimum bids for the unsold frequency blocks is EUR 60 million. The resulting total amount is therefore EUR 930 million.

Table 18: Combinatorial package bids in the principal stage – Example 5

Bidder	Package									Package bid (principal stage)
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	0	1	1	0	0	2	EUR 320 million
	1	1	0	1	1	0	0	0	2	EUR 250 million
Ben	0	2	0	0	2	1	1	4	0	EUR 320 million
	0	2	0	0	2	1	1	3	0	EUR 295 million
	0	1	0	0	2	1	0	4	0	EUR 185 million
	0	1	0	0	2	1	0	3	0	EUR 170 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 310 million
	0	0	0	0	2	0	0	4	2	EUR 300 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

Steps a) and b): for each winning bidder, calculate the total amount which would result from the winner determination procedure if that bidder had not submitted a bid. Determine the maximum bid reduction.

Again, the maximum discounts possible will be calculated as described in the previous examples. The maximum discounts resulting from this calculation as well as the potential base prices are summarised in Table 19.

Table 19: Potential base prices – Example 5

Bidder	Next best maximum total amount	Maximum discount	Potential base price
Alan	EUR 830 m	EUR 930 m - EUR 830 m = EUR 100 m	EUR 150 m
Ben	EUR 840 m	EUR 930 m - EUR 840 m = EUR 90 m	EUR 230 m
Fred	EUR 910 m	EUR 930 m - EUR 910 m= EUR 20 m	EUR 280 m

Step c): we then generate a list of constraints and initialise it with the conditions that the individual discounts must not be higher than the maximum discounts calculated in the previous step, and that the overall sum of discounts over all bidders together must not be larger than the reduction in the total bid amount if the bidders had not taken part in the auction.

If the winning bidders had not taken part in the auction, then the next best combination yielding the highest total revenues would be to allow Greg and Emma to win their respective package bids. In this allocation one frequency block in each of the categories A1, A3, B1 and B3 as well as three frequency blocks in A2 and B2 and two frequency blocks in C1 would remain unsold. The total allocation amount would be EUR 630 million; the sum of all discounts over all winners would thus be EUR 300 million (EUR 930 million – EUR 630 million).

The constraints, resulting from Table 19, are as follows:

$$d_{Alan} \geq 0$$
$$d_{Ben} \geq 0$$
$$d_{Fred} \geq 0$$
$$d_{Alan} \leq 100Mio$$
$$d_{Ben} \leq 90Mio$$
$$d_{Fred} \leq 20Mio$$
$$d_{\{Alan,Ben,Fred\}} \leq 300Mio$$

Step d): maximise the sum of the individual discounts subject to these constraints.

Maximising the sum of discounts under the given constraints yields the following solution:

$$d^{\star}_{Alan} = 100Mio$$
$$d^{\star}_{Ben} = 90Mio$$
$$d^{\star}_{Fred} = 20Mio$$

Steps e), f) and g): reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero. Then determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and the resulting total amount, and compare these with the original winning bids using the reduced bid amounts.

We subtract the discounts from the winners' bids indicated in Table 18, thereby arriving at the modified bids indicated in Table 20.

Table 20: Modified bids, first iteration – Example 5

Bidder	**Package**									**Modified bid (after application of discount)**
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	0	1	1	0	0	2	EUR 220 million
	1	1	0	1	1	0	0	0	2	EUR 150 million
Ben	0	2	0	0	2	1	1	4	0	EUR 230 million
	0	2	0	0	2	1	1	3	0	EUR 205 million
	0	1	0	0	2	1	0	4	0	EUR 95 million
	0	1	0	0	2	1	0	3	0	EUR 80 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 290 million
	0	0	0	0	2	0	0	4	2	EUR 280 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

Again we arrive at the combination (of the now modified bids) yielding the highest total revenues. This combination is highlighted in blue in Table 20. In this combination one

frequency block in each of the categories A1, A3, B1 and B3 as well as three frequency blocks in A2 and B2 and two frequency blocks in C1 would remain unsold. The sum of the modified bids amounts to EUR 550 and the sum of the unsold frequency blocks to EUR 220 million. Therefore, the modified total amount resulting from this combination equals EUR 770 million. The sum of the modified original winning bids amounts to EUR 660 million and the total of the minimum bids for the originally unsold frequency blocks to EUR 60 million. The modified total amount of the original winning combination of bidders (EUR 720 million) differs therefore from the highest total amount achievable with the modified bids (EUR 770 million).

Compared to the original result, Alan and Ben are eliminated from the modified winning combination of bidders.

Step h): if there are winning bidders from the original winner determination procedure that are no longer among the winning bidders with the modified bids, then expand the list of constraints to include the condition that the sum of bid reductions for those bidders that are eliminated from the winning combination must not be higher than the reduction of the total bid amount if those bidders had not taken part in the auction. Proceed to Step d).

We therefore have to find the next best combination on the basis of the original bids submitted which would have arisen if Alan and Ben had not participated in the auction. In this allocation Fred's original winning bid is successful and Greg is awarded his first package. The sum of the winning bids in this allocation is EUR 570 million. In this allocation one frequency block in each of the categories A1, A3, B1 and B3 as well as three frequency blocks in A2 and B2 and two frequency blocks in C1 remain unsold. The total of the minimum bids for any frequency blocks unsold in this allocation therefore amounts to EUR 220 million and the total amount to EUR 790 million. Therefore, $\sigma(Alan, Ben) = 930Mio - 790Mio = 140Mio$. Accordingly, the maximum sum of discounts for Alan and Ben taken together can be at most EUR 140 million. We therefore add the following constraint to our list:

$$d_{Alan} + d_{Ben} \leq 140Mio$$

Step d): maximise the sum of the individual discounts subject to these constraints.

One possible solution to the maximisation problem given the expanded list of constraints is as follows:

$$d^{\star}_{Alan} = 100Mio$$
$$d^{\star}_{Ben} = 40Mio$$
$$d^{\star}_{Fred} = 20Mio$$

This is only one possible solution, as any combination of non-negative discounts for Alan and Ben that meet the condition $d_{Alan} + d_{Ben} = 140Mio$ (and, of course, the other constraints) maximises the sum of discounts.

Step e): reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero.

Subtracting these discounts from the winners' bids in Table 18 yields the modified winning bids indicated in the table below.

Table 21: Modified package bids, second iteration – Example 5

Bidder	Package									Modified bid (after application of discount)
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	0	1	1	0	0	2	EUR 220 million
	1	1	0	1	1	0	0	0	2	EUR 150 million
Ben	0	2	0	0	2	1	1	4	0	EUR 280 million
	0	2	0	0	2	1	1	3	0	EUR 255 million
	0	1	0	0	2	1	0	4	0	EUR 145 million
	0	1	0	0	2	1	0	3	0	EUR 130 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 290 million
	0	0	0	0	2	0	0	4	2	EUR 280 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

Steps f) and g): then determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and the resulting total amount, and compare these with the original winning bids using the reduced bid amounts.

Following the determination of the maximised total revenues on the basis of these modified bids, it becomes evident that the modified maximised total revenues resulting from this calculation (Step f) is not any higher than the modified total amount of the original winning combination of bidders, including the total of the minimum bids for the unsold frequency blocks, i.e. EUR 770 million.

Therefore, the original winning combination of bidders remains the optimum (even if another combination – i.e. the combination of Fred's and Greg's bids – yields the same value), and we can thus proceed to the quadratic optimisation.

Steps i), j) and k): minimise the sum of squared differences between the individual discounts and the individual maximum discounts $\sigma(\{j\}) = (V - V^{-j})$ subject to the constraints and the condition that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts. Reduce the bid amounts for all of the winners' bids by their corresponding discounts; if this yields a negative bid amount, set this bid amount to zero. Determine the winning bids in accordance with Rule 5.10 using the modified bid amounts and calculate the resulting total amount.

The solution to the quadratic minimisation program subject to the constraints is as follows:

$$d^{\star}_{Alan} = 75Mio$$
$$d^{\star}_{Ben} = 65Mio$$
$$d^{\star}_{Fred} = 20Mio$$

We reduce all of the winners' bids by the discounts found, to obtain the modified bids listed in Table 22.

Table 22: Modified bids, third iteration – Example 5

Bidder	Package									Modified bid (after application of discount)
	A1	A2	A3	B1	B2	B3	C1	C2	C3	
Alan	1	2	0	0	1	1	0	0	2	EUR 245 million
	1	1	0	1	1	0	0	0	2	EUR 175 million
Ben	0	2	0	0	2	1	1	4	0	EUR 255 million
	0	2	0	0	2	1	1	3	0	EUR 230 million
	0	1	0	0	2	1	0	4	0	EUR 120 million
	0	1	0	0	2	1	0	3	0	EUR 105 million
Doris	0	0	0	0	2	0	0	4	0	EUR 110 million
Emma	0	0	0	0	2	0	0	4	2	EUR 140 million
Fred	0	0	0	0	2	0	0	5	2	EUR 290 million
	0	0	0	0	2	0	0	4	2	EUR 280 million
Greg	0	1	0	0	0	0	0	4	3	EUR 270 million
	1	1	0	0	0	0	0	0	1	EUR 100 million

Step l): If the original winning bids using the reduced bid amounts do not yield a lower value than the total amount calculated in Step k), then the discounts found are the relevant discounts and the base prices are equal to the winning bids reduced by those amounts.

The resulting base prices are shown in Table 23.

Table 23: Base prices for winning bids – Example 5

Bidder	Base price
Alan	EUR 175 million
Ben	EUR 255 million
Fred	EUR 280 million
Total	EUR 710 million

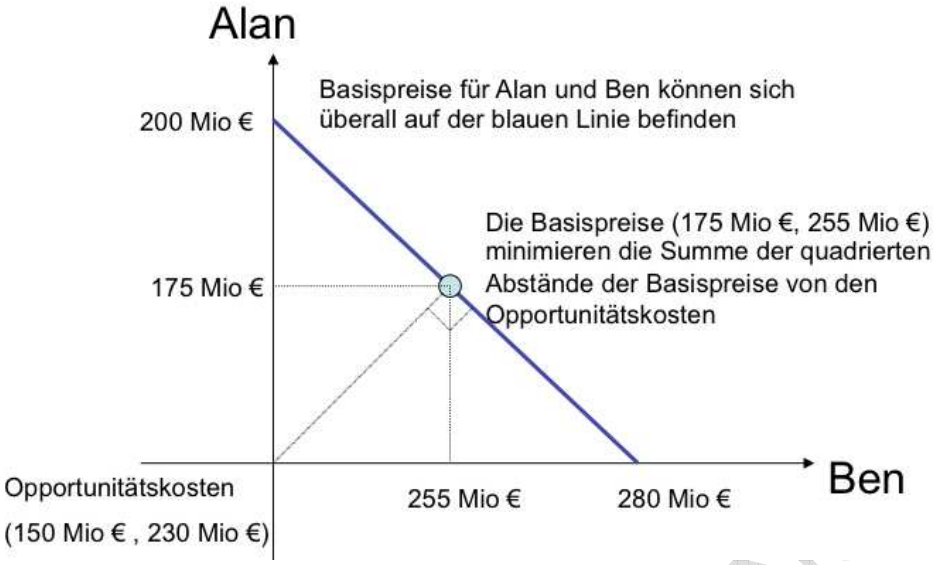
The intuitive explanation for this solution is as follows:

The winning combination of bidders would leave unsold frequency blocks of EUR 60 million as measured at their minimum bid value. If Alan and Ben had not participated in the auction, Greg would have been awarded his first package (as well as Fred, who would receive the same package as in the winning combination of bidders). This allocation would have left unsold frequency blocks of EUR 220 million as measured at their minimum bid value. This means that additional frequency blocks amounting to EUR 160 million would not have been sold, if Alan and Ben had not participated in the auction. The sum of the base prices for Alan and Ben must be high enough to outbid Greg (EUR 270 million) and to compensate for the total of the minimum bids for the otherwise unsold frequency blocks (EUR 160 million).

In this example, the sum of the opportunity costs of Alan's and Ben's packages amounts to EUR 380 million (EUR 150 million + EUR 230 million). However, this is less than Greg would be willing to pay for this combination (EUR 270 million) and the total of the minimum bids for the additionally unsold frequency blocks (EUR 160 million). Ben and Alan thus have to pay the additional EUR 50 million together. There are various ways to split this amount between them. The figure below shows all combinations of base prices for Alan and Ben which (taken

together) are as high as Greg's bid and the total of the minimum bids for the additionally unsold frequency blocks (blue line in Figure 1).

Figure 1: Minimisation of squared differences – Example 5



In order to determine a unique set of base prices, the sum of squared distances of base prices from opportunity costs is minimised. This selection criterion always identifies a unique set of base prices.

Appendix C: Determination of assignment options

The auction software provides the bidders with all possible assignment options. These options must meet the conditions set out in Rules 6.2 and 6.3.

The following examples illustrate the determination of assignment options.

Example 6: Determination of assignment options for C1, C2 and C3 during the transition phase 2013-2019

Initially, the assignment of the frequencies in the C1, C2 and C3 categories is determined for the period from 2013 to 2019. We assume that four bidders have acquired frequencies in this band and that one frequency block remains unsold:

Bidder	C1	C2	C3
Alan	1		
Ben		2	3
Carl		2	2
Doris		4	
Unsold	1		

Category C1: Alan is the only winner of frequencies in the C1 category. Therefore, no separate assignment stage is required for C1. Alan will automatically receive the specific frequency block LC02, leaving the unsold frequencies at the lower end of the category (frequency block LC01).

Category C2: Ben, Carl and Doris have acquired frequencies in the C2 category. The assignment of specific frequencies is only subject to the constraint that the number of specific frequency blocks assigned to each bidder exactly matches the number of abstract frequency blocks the respective bidder has acquired in the auction. It cannot be guaranteed that contiguous frequencies will be assigned. Therefore, any one of 28 possible combinations of two specific frequency blocks each is a permissible assignment option for Ben and Carl.

- LC03+LC04, LC03+LC05, LC03+LC06, LC03+LC07, LC03+LC08, LC03+LC09, LC03+LC10
- LC04+LC05, LC04+LC06, LC04+LC07, LC04+LC08, LC04+LC09, LC04+LC10
- LC05+LC06, LC05+LC07, LC05+LC08, LC05+LC09, LC05+LC10
- LC06+LC07, LC06+LC08, LC06+LC09, LC06+LC10
- LC07+LC08, LC07+LC09, LC07+LC10
- LC08+LC09, LC08+LC10
- LC09+LC10

For Doris, who has acquired three abstract frequency blocks in the C2 category, the following 56 permissible assignment options exist.

- LC03+LC04+LC05, LC03+LC04+LC06, LC03+LC04+LC07, LC03+LC04+LC08, LC03+LC04+LC09, LC03+LC04+LC10

- LC03+LC05+LC06, LC03+LC05+LC07, LC03+LC05+LC08, LC03+LC05+LC09, LC03+LC05+LC10
- LC03+LC06+LC07, LC03+LC06+LC08, LC03+LC06+LC09, LC03+LC06+LC10
- LC03+LC07+LC08, LC03+LC07+LC09, LC03+LC07+LC10
- LC03+LC08+LC09, LC03+LC08+LC10
- LC03+LC09+LC10
- LC04+LC05+LC06, LC04+LC05+LC07, LC04+LC05+LC08, LC04+LC05+LC09, LC04+LC05+LC10
- LC04+LC06+LC07, LC04+LC06+LC08, LC04+LC06+LC09, LC04+LC06+LC10
- LC04+LC07+LC08, LC04+LC07+LC09, LC04+LC07+LC10
- LC04+LC08+LC09, LC04+LC08+LC10
- LC04+LC09+LC10
- LC05+LC06+LC07, LC05+LC06+LC08, LC05+LC06+LC09, LC05+LC06+LC10
- LC05+LC07+LC08, LC05+LC07+LC09, LC05+LC07+LC10
- LC05+LC08+LC09, LC05+LC08+LC10
- LC05+LC09+LC10
- LC06+LC07+LC08, LC06+LC07+LC09, LC06+LC07+LC10
- LC06+LC08+LC09, LC06+LC08+LC10
- LC06+LC09+LC10
- LC07+LC08+LC09, LC07+LC08+LC10
- LC07+LC09+LC10
- LC08+LC09+LC10

Category C3: Ben has acquired three frequency blocks and Carl two frequency blocks in the C3 category. Both bidders are offered the assignment options corresponding to the number of frequency blocks won in the auction and permitting consecutive assignment of frequencies to the other winning bidder. Therefore, the assignment options are as follows:

- Ben: LC11+LC12+LC13 or LC13+LC14+LC15
- Carl: LC14+LC15 or LC11+LC12

Example 7: Determination of assignment options in the categories A1, A2, A3 and B1, B2, B3, as well as C1, C2, C3 from 2020 onwards

In a second assignment stage, the frequency blocks in all categories will be assigned for the period from 2020 onwards. Again we assume that there are four winners and several unsold frequency blocks. The bidders have acquired the following frequency blocks in the pre-auction and in the principal stage:

Bidder	A1	A2	A3	B1	B2	B3	C1	C2	C3
Alan	1	1		1	1		1		
Ben		2			2			2	3
Carl		1	1		1			2	2
Doris						1		4	
Unsold					1		1		

800 MHz band (categories A1, A2 and A3): Alan's frequencies are assigned automatically. since Alan has acquired the frequency block in the A1 category and one frequency block in the A2 category, he will be assigned the frequency blocks LA01 and LA02 automatically in accordance with Rule 6.3.1 in order to guarantee a contiguous assignment. Carl acquires two frequency blocks in A2/A3 and Ben acquires two frequency blocks in A2. None of the frequency blocks in this band remains unsold. This means the frequency blocks LA03, LA04, LA05 and LA06 are still available for Ben and Carl. Due to the fact that only contiguous assignments are allowed, the following assignment options are available:

- Ben: LA03 and LA04 or LA05 and LA06
- Carl: LA03 and LA04 or LA05 and LA06

900 MHz band (categories B1, B2 and B3): Alan's contiguous assignment (LB01 and LB02) is generated automatically. Doris has acquired the specific frequency block B3, so her assignment is also clearly determined. She receives LB07. Ben and Carl together have won three frequency blocks in the B2 category. One frequency block in this category remains unsold. According to Rule 6.3.4, the unsold abstract frequency block will be placed at the upper end of the available frequency range (i.e. LB06). This means the frequency blocks LB03, LB04 and LB05 are still available for Ben and Carl. Due to the fact that only contiguous assignments are allowed, the following assignment options are available:

- Ben: LB03 and LB04 or LB04 and LB05
- Carl: LB03 or LB05

1800 MHz band (categories C1, C2 and C3): For the period from 2020 onwards, any frequency blocks acquired by the bidders in the various categories are added up and the entire band is regarded as a single category C with a total of 15 frequency blocks (see Rule 6.3.7). There are four winners and one unsold frequency block in this category:

- Alan: one frequency block
- Ben: five frequency blocks
- Carl: four frequency blocks
- Doris: four frequency blocks
- Unsold: one frequency block

In contrast to the assignment for the period 2013-2019, only contiguous assignment options are available for the period from 2020 onwards. Unsold frequencies must be placed contiguously as well. The unsold frequencies need not be placed at the upper end of the band, but may be located throughout the band, as long as this allows contiguous assignment to the other winning bidders.

Alan can submit supplementary bids for any positions within the band allowing a contiguous assignment of frequencies to the other winners and to the placeholder. This means, he *cannot* bid for LC03, for instance, because this would make it impossible to assign contiguous frequencies to the other winning bidders. Therefore, the following 10 options are available to Alan:

- LC01, LC02, LC05, LC06, LC07, LC09, LC10, LC11, LC14, LC15

Ben has the following nine permissible assignment options:

- LC01-LC05, LC02-LC06, LC03-LC07, LC05-LC09, LC06-LC10, LC07-LC11, LC09-LC13, LC10-LC14, LC11-LC15

Carl and Doris have acquired four frequency blocks each, and therefore can accordingly bid for the same ten assignment options:

- LC01-LC04, LC02-LC05, LC03-LC06, LC05-LC08, LC06-LC09, LC07-LC10, LC08-LC11, LC10-LC13, LC11-LC14, LC12-LC15

The unsold frequency block may be placed at the following ten positions within the band:

- LC01, LC02, LC05, LC06, LC07, LC09, LC10, LC11, LC14, LC15

The assignment options available to the winners as well as the unsold frequencies generate a total of 120 different bandplans. The bids submitted in the assignment stage determine which one of these bandplans will ultimately be implemented.

Appendix D: Calculation of top-up prices

Description of the algorithm used to calculate top-up prices in accordance with Rule 6.7.

The procedure described below finds a unique set of top-up prices which meet the conditions of Rule 6.7. The procedure is essentially similar to the one used to calculate base prices, except for the fact that exactly one assignment bid from each bidder must be accepted.

Let W be the set of bidders participating in the assignment stage (i.e. the bidders that placed winning bids in the principal stage). The winner determination procedure in accordance with Rule 6.6 determines the sum of the winning bids v and the assignment option valid for each bidder. v^{-C} denotes the sum of the winning assignment bids identified in the winner determination procedure if the bidders in $C \subseteq W$ do not express any preferences for the assignment of certain blocks, that is, if all of the assignment bids submitted by these bidders show a bid amount of zero. $\sigma(C) = (v - v^{-C})$ denotes the decline in the total assignment bid value if the bidders in $C \subseteq W$ do not submit any positive assignment bids.

a) Generate a list of constraints B for a restricted optimisation problem and initialise it as $B = \left\{ \forall j \in W : d_j \leq \sigma(\{j\}), \sum_{j \in W} d_j \leq \sigma(W) \right\}$.

b) Solve the following maximisation problem: $\max \sum_{j \in W} d_j$ subject to the constraints in B and the constraint that the individual values of d_j cannot be negative. In general, many solutions to this maximisation problem may exist. If this is the case, pick a random solution d^*.

c) Reduce the bid amounts for all assignment bids submitted by bidder j by d_j^*; if this yields a negative bid amount, set this bid amount to zero.

d) Determine the winning assignment bids in accordance with Rule 6.6 using the modified bid amounts and calculate the corresponding total winning bid amount.

e) If the original winning assignment bids using the reduced bid amounts do not yield a lower value than the total bid amount calculated in Step d), then the maximum discount which can be granted to all winning assignment bidders is therefore, $D^* = \sum_{j \in W} d_j^*$. Proceed to Step g).

f) If there are winning assignment bidders from the original assignment winner determination procedure whose assignment option given the modified assignment bids is not the same as the option resulting from the original assignment bids and whose modified assignment bid is not positive,[8] then add the following constraint to the list of constraints in B:

[8] This reflects the fact that the "losers" are those bidders whose preferences for certain assignment options are not taken into account. This means that, where bidders receive a different assignment option with a modified assignment bid of zero compared to their original assignment bid, those bidders were granted too large a discount.

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders that receive a different assignment option and whose modified assignment bid is not positive. Proceed to Step b).

g) Solve the following minimisation problem: $\min \sum_{j \in W} \left(d_j - \sigma(\{j\})\right)^2$ subject to the constraints in B, the constraint that the individual values of d_j cannot be negative, and the constraint that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts, i.e. $\sum_{j \in W} d_j = D^*$. Let d^{**} be the solution to this minimisation problem.

h) Reduce the bid amounts for all assignment bids submitted by Bidder j by d_j^{**}; if this yields a negative bid amount, set this bid amount to zero.

i) Determine the winning assignment bids in accordance with Rule 6.6 using the modified bid amounts and calculate the corresponding total winning bid amount.

j) If the original winning assignment bids at the reduced bid amounts do not yield a lower value than the total bid amount calculated in Step i), then the values of d_j^{**} are the relevant discounts and the top-up prices are equal to the winning assignment bids reduced by these amounts, that is, $p_j = b_j - d_j^{**}$ for every $j \in W$, with b_j denoting the bidder's winning assignment bid.

k) If there are winning assignment bidders from the original assignment winner determination procedure whose assignment option given the modified assignment bids is not the same as the option resulting from the original assignment bids and whose modified assignment bid is not positive, then add the following constraint to the list of constraints in B:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders that receive a different assignment option and whose modified assignment bid is not positive. Proceed to Step g).

Due to numerical inaccuracies in the automated calculation of top-up prices (minimum revenue core prices), minimal deviations from the theoretical top-up price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated top-up price. In very rare cases, the top-up price calculated may be a few euros higher than the theoretical top-up price.

Usually, the results should be available in less than one second. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results, or additional processing capacity may be required. In such an improbable case, the submission of bids will be concluded and the results will be announced at a later point in time.

Example 8: Calculation of top-up prices

In order to simplify the process, the calculation of top-up prices in the assignment stage will only be discussed with respect to the 1800 MHz band for the period from 2020 onwards. The assignment of the other bands and of the 1800 MHz band for the period 2013-2019 is determined in an analogous manner.

This example is calculated with different winning bidders than the previous example, which was based on four winning bidders and one unsold frequency block. This alone generates 120 possibilities for different bandplans. This would make the calculation of top-up prices for this example unnecessarily complicated.[9] Furthermore, unsold frequency blocks in the 1800 MHz band must be assigned contiguously from 2020 onwards, which can be illustrated by way of example only if several frequency blocks remain unsold.

We therefore assume that there are three winning bidders and two unsold frequency blocks:

Bidder	C1	C2	C3
Ben		2	3
Carl		2	2
Doris		4	
Unsold	2		

For the period from 2020 onwards the three categories C1, C2 and C3 will be integrated into one single category. Any frequency blocks acquired by the winning bidders in the three categories will be added up for the period from 2020:

Bidder	Category C
Ben	5
Carl	4
Doris	4
Unsold	2

From 2020 onwards, the assignment options in the 1800 MHz band for the various winning bidders are as follows:

- Ben: LC01-LC05, LC03-LC07, LC05-LC09, LC07-LC11, LC09-LC13, LC11-LC15
- Carl: LC01-LC04, LC03-LC06, LC05-LC08, LC06-LC09, LC07-LC10, LC08-LC11, LC10-LC13, LC12-LC15
- Doris: LC01-LC04, LC03-LC06, LC05-LC08, LC06-LC09, LC07-LC10, LC08-LC11, LC10-LC13, LC12-LC15
- Unsold: LC01-LC02, LC05-LC06, LC06-LC07, LC09-LC10, LC10-LC11, LC14-LC15

There are several potential bandplans available to each of the bidders.

[9] The number of possible bandplans is calculated as *n!, where n* is the number of winning bidders, or as *(n+1)!* if there is at least one unsold frequency block.

Figure 2: Potential positions in the bandplan for Ben

LC01	LC02	LC03	LC04	LC05	LC06	LC07	LC08	LC09	LC10	LC11	LC12	LC13	LC14	LC15

Figure 3: Potential positions in the bandplan for Carl and Doris

LC01	LC02	LC03	LC04	LC05	LC06	LC07	LC08	LC09	LC10	LC11	LC12	LC13	LC14	LC15

Figure 4: Potential positions of unsold frequency blocks in the bandplan

LC01	LC02	LC03	LC04	LC05	LC06	LC07	LC08	LC09	LC10	LC11	LC12	LC13	LC14	LC15

From the assignment options of the individual winning bidders and the unsold frequencies, 24 possibilities for complete bandplans result:

Figure 5: Different possibilities of complete bandplans (B=Ben, C=Carl, D=Doris)

	LC01	LC02	LC03	LC04	LC05	LC06	LC07	LC08	LC09	LC10	LC11	LC12	LC13	LC14	LC15
1			B	B	B	B	B	C	C	C	C	D	D	D	D
2			B	B	B	B	B	D	D	D	D	C	C	C	C
3			C	C	C	C	B	B	B	B	B	D	D	D	D
4			D	D	D	D	B	B	B	B	B	C	C	C	C
5			C	C	C	C	D	D	D	D	B	B	B	B	B
6			D	D	D	D	C	C	C	C	B	B	B	B	B
7	C	C	C	C			B	B	B	B	B	D	D	D	D
8	D	D	D	D			B	B	B	B	B	C	C	C	C
9	C	C	C	C			D	D	D	D	B	B	B	B	B
10	D	D	D	D			C	C	C	C	B	B	B	B	B
11	B	B	B	B	B			C	C	C	C	D	D	D	D
12	B	B	B	B	B			D	D	D	D	C	C	C	C
13	C	C	C	C	D	D	D	D			B	B	B	B	B
14	D	D	D	D	C	C	C	C			B	B	B	B	B
15	B	B	B	B	B	C	C	C	C			D	D	D	D
16	B	B	B	B	B	D	D	D	D			C	C	C	C
17	C	C	C	C	B	B	B	B	B			D	D	D	D
18	D	D	D	D	B	B	B	B	B			C	C	C	C
19	B	B	B	B	B	C	C	C	C	D	D	D	D		
20	B	B	B	B	B	D	D	D	D	C	C	C	C		
21	C	C	C	C	B	B	B	B	B	D	D	D	D		
22	D	D	D	D	B	B	B	B	B	C	C	C	C		
23	C	C	C	C	D	D	D	D	B	B	B	B	B		
24	D	D	D	D	C	C	C	C	B	B	B	B	B		

We assume that the bidders submit the following assignment bids (zero bids are added automatically by the auction software).

Table 24: Assignment bids – Example 8

Bidder	Assignment option	Assignment bid	Possible complete bandplans
Ben	LC01-LC05	EUR 0.5 million	11,12,15,16,19,20
	LC03-LC07	EUR 0.2 million	1,2
	LC05-LC09	0	17,18,21,22
	LC07-LC11	0	3,4,7,8
	LC09-LC13	0	23,24
	LC11-LC15	EUR 0.1 million	5,6,9,10,13,14
Carl	LC01-LC04	EUR 0.45 million	7,9,13,17,21,23
	LC03-LC06	EUR 0.1 million	3,5
	LC05-LC08	EUR 0.08 million	14,24
	LC06-LC09	0	15,19
	LC07-LC10	0	6,10
	LC08-LC11	EUR 0.05 million	1,11
	LC10-LC13	0	20,22
	LC12-LC15	0	2,4,8,12,16,18
Doris	LC01-LC04	EUR 0.1 million	8,10,14,18,22,24
	LC03-LC06	EUR 0.05 million	4,6
	LC05-LC08	0	13,23
	LC06-LC09	0	16,20
	LC07-LC10	0	5,9
	LC08-LC11	0	2,12
	LC10-LC13	0	19,21
	LC12-LC15	EUR 0.3 million	1,3,7,11,15,17

Similar to the winner determination procedure in the principal stage, the combination yielding the highest total amount will be determined from the assignment bids (see Rule 6.6). It is therefore calculated for which of the possible bandplans the highest total amount was bid. The corresponding winning bids are highlighted in blue. This combination applies to bandplan number 11.

The calculation of top-up prices using the modified second-price rule is similar to the calculation of base prices in the principal stage. The difference is that in the calculation of opportunity costs (or the maximum discounts according to Step a)), not all of a bidder's assignment bids are eliminated; instead, the bid amounts are set to the respective minimum bid (in this case zero). The reason for this difference is that the opportunity costs are determined by a bidder's preference for certain specific frequency blocks and not by the bidder's demand for frequency blocks in general. The reduction of all bids to the respective minimum bids reflects the case in which a bidder expresses no preferences for specific frequency blocks.

If all of Ben's assignment bids are set to zero, then bandplan 7 or 17 would yield a higher amount than bandplan 11. Therefore, if Ben did not have any preference regarding the assignment option, Carl would acquire the frequency blocks LC01-LC04 and Doris the frequency blocks LC12-LC15. In this case, the total amount would be (EUR 0.45 million + EUR 0.3 million) EUR 0.75 million. This means that the maximum discount (according to

Step a)) for Ben is EUR 0.1 million and the opportunity costs arising from Ben's preferences is EUR 0.4 million (EUR 0.5 million - EUR 0.1 million). An analogous procedure is carried out for the other bidders. The results are summarised in the table below.

Table 25: Calculation of top-up prices – Example 8

Bidder	Frequency blocks	Modified maximum total amount	Maximum discount	Opportunity costs (potential top-up price)
Ben	LC01-LC05	EUR 0.75 m	EUR 0.85 m - EUR 0.75 m = EUR 0.1 m	EUR 0.5 m - EUR 0.1 m = EUR 0.4 m
Carl	LC08-LC11	EUR 0.8 m	EUR 0.85 m - EUR 0.8 m = EUR 0.05 m	EUR 0.05 m - EUR 0.05 m = EUR 0
Doris	LC12-LC15	EUR 0.55 m	EUR 0.85 m - EUR 0.55 m = EUR 0.3 m	EUR 0.3 m - EUR 0.3 m = EUR 0

The linear optimisation (Step b) yields discounts in the maximum amounts according to Table 25. Subtracting these discounts from the bids yields the modified assignment bids indicated in Table 26 (Step c). The original combination of winning bidders still maximises the revenues from the auction (Steps d) and e)), that is, the top-up prices are equal to the opportunity costs. This is illustrated in the table below.

Table 26: Modified assignment bids – Example 8

Bidder	Assignment option	Assignment bid	Possible complete bandplans
	LC01-LC05	EUR 0.4 million	11,12,15,16,19,20
	LC03-LC07	EUR 0.1 million	1,2
	LC05-LC09	0	17,18,21,22
	LC07-LC11	0	3,4,7,8
	LC09-LC13	0	23,24
Ben	LC11-LC15	0	5,6,9,10,13,14
	LC01-LC04	EUR 0.4 million	7,9,13,17,21,23
	LC03-LC06	EUR 0.05 million	3,5
	LC05-LC08	EUR 0.03 million	14,24
	LC06-LC09	0	15,19
	LC07-LC10	0	6,10
	LC08-LC11	0	1,11
	LC10-LC13	0	20,22
Carl	LC12-LC15	0	2,4,8,12,16,18
	LC01-LC04	0	8,10,14,18,22,24
	LC03-LC06	0	4,6
	LC05-LC08	0	13,23
	LC06-LC09	0	16,20
	LC07-LC10	0	5,9
	LC08-LC11	0	2,12
	LC10-LC13	0	19,21
Doris	LC12-LC15	0	1,3,7,11,15,17

The top-up prices of the assignment stage calculated in this way are indicated in Table 27.

Table 27: Auction results – Example 8

Bidder	Package acquired	Top-up price
Ben	LC01-LC05	EUR 0.4 m
Carl	LC08-LC11	EUR 0
Doris	LC12-LC15	EUR 0

Appendix E: Algorithm for determining winners

The purpose of the winner determination procedure is to identify the combination of bids which yields the highest total bid amount, consisting of the sum of the bid amounts and the minimum bid value of any unsold frequency blocks (only the sum of the bid amounts in the assignment stage), subject to accepting at most one bid per bidder (in the assignment stage: exactly one bid per bidder) and that no more frequency blocks are assigned than are available.

The approach we use is based on the insight that any optimal distribution of the available frequency blocks among the winning bidders must distribute subsets of frequency blocks optimally among subsets of winning bidders. This allows us to split the winner determination procedure into sub-problems and to solve these sequentially. One suitable approach for this purpose is as follows:

- Identify the set $\boldsymbol{T}$ of all possible subsets of block combinations given the overall set of frequency blocks L.
- For each $t \in T$ calculate the minimum bid total for any frequency blocks included in this set $v_0(t)$ (for the assignment stage, set $v_0(t)$ to zero).
- For all $n = 1,...,N$ bidders: for each $t \in T$, identify the highest total amount $v_n(t)$ that can be achieved, if the corresponding block combination were made available to the initial bidders n. The maximum total amount results from the optimal decision of the nth bidder combined with the maximum value that can be achieved if the remaining frequency blocks $(t-b)$ are made available to the remaining $n-1$ (i. e. $v_{n-1}(t-b)$) bidders, where b is the block combination in the best bid made by the nth bidder. Here $v_0(t-b)$ is the total of minimum bids for the remaining frequency blocks $(t-b)$ (in the assignment stage this will be zero).
- The maximum total amount is then equal to $v_N(L)$.
- Determine the optimal bid $v_N(L)$ of the Nth bidder contained in b_N^*.
- For the remaining $n = N-1,...,1$ bidders: identify the optimal bid for Bidder n using $v_n(L - \sum_{i=n+1}^{N} b_i^*)$.