Telekom-Control-Kommission
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F 4/08

Vienna, June 28, 2010

Rules of Procedure under § 55 Par. 9 TKG 2003
Auction Procedure for Frequency Assignments in the 2.6 GHz Band
(non-binding Web version)

NON-BINDING-TRANSLATION

1. General

The purpose of this procedure is to assign frequency blocks in the 2.6 GHz band, which includes both paired and unpaired spectrum. The auction procedure will be carried out in the form of a combinatorial clock auction. This procedure consists of two stages (separate auctions): a principal stage that determines the number of paired and unpaired lots to be acquired by each bidder, and an assignment stage in which the specific frequency blocks are assigned to the winners of paired and unpaired spectrum.

The first stage (principal stage) consists of a number of open rounds of bidding (clock stage) in which bidders submit combinatorial package bids for abstract frequency blocks. In their bids, bidders indicate the desired number of abstract frequency packages they would like to acquire at the current prices in each round. Once the clock stage has been completed, bidders may submit supplementary package bids for (other) combinations of abstract frequency blocks in the course of a sealed-bid stage. The winning combination of bids which maximises the auction revenues will then be determined algorithmically from all bids submitted during the principal stage; at most one bid from each bidder (from all bids submitted during the clock stage and the sealed-bid stage) will be included in the winning combination.

The winning bidders will be those whose bids are included in the successful combination which maximises the revenues from the auction. The winning bidders will receive the number of abstract frequency blocks indicated in their winning bids in each category at the respective "base prices," which will be determined using a modified second price rule. These prices represent the lowest prices the winning bidders would have had to offer (jointly) in order for their respective bids to be successful (minimum core prices). Moreover, the base price of each winning bid must be at least as high as the sum of minimum bids (start bid or reserve prices) for the abstract frequency blocks included in the respective package.

The second stage of the auction (assignment stage) consists of a single sealed-bid round of bidding in which winners of the principal stage can submit package bids on various combinations of actual contiguous frequency blocks which are compatible with the number of abstract frequency blocks won by the successful bidders in the principal stage. The winning bidders will receive the actual frequency blocks as indicated in their winning bids in each category at the respective "top-up" prices. Top-up prices will also be determined on the basis of a modified second price rule.

The total price to be paid is calculated as the sum of the prices from the two stages of the auction.

Those applicants who are not excluded from the frequency assignment procedure in accordance with Art. 55 Par. 8 TKG 2003 will be admitted to the principal stage of the auction. Those participants who place winning bids and thus acquire abstract frequency blocks in the principal stage will then be admitted to the assignment stage of the auction.

The maximum amount of spectrum which a bidder can acquire is limited by the eligibility requested by that bidder, and by the spectrum caps defined by the Telekom-Control-Kommission (cf. Sections 3.3 and 3.4 of the Tender Documentation).

The auctioneer will be the Telekom-Control-Kommission or a member appointed by the Commission. The Telekom-Control-Kommission may also appoint employees of RTR's Telecommunications Division to carry out the auction.

2. Collusion and discontinuation of the procedure

2.1. Collusion

Any form of cooperation among the applicants or their stakeholders, be it direct or indirect, with the intention of influencing the events or results of the auction (collusive behaviour) is prohibited. Should applicants cooperate in a collusive manner before or during the auction procedure, they may be excluded from the remainder of the procedure (Art. 55 Par. 9 TKG 2003). The auctioneer shall have the right to take all appropriate measures to prevent collusive behaviour.

Likewise, threats to competitors or the public announcement of participation in the auction, or of bids or bidding strategies – also prior to the auction procedure – may result in exclusion from the assignment procedure.

In this context, specific reference is also made to the provisions of general competition law in Austria.

2.2. Discontinuation of the procedure

The Telekom-Control-Kommission shall have the right to discontinue the auction procedure if it discovers collusive behaviour among applicants and an efficient, fair and non-discriminatory procedure cannot be guaranteed (Art. 55 Par. 12 No. 1 TKG 2003), or if other significant reasons are found which endanger the proper execution of the frequency assignment procedure. Under these circumstances, the Telekom-Control-Kommission will decide whether the procedure is to be discontinued in accordance with Section 5.9 of the Tender Documentation or a new auction date is to be set.

3. Objects of the auction

3.1. Eligibility to participate

3.1.1. Those applicants who are not excluded from the frequency assignment procedure in accordance with Art. 55 Par. 8 TKG 2003 will be admitted to the auction.

3.2. Objects of the auction

3.2.1. The available frequency blocks (cf. Section 2.2 of the Tender Documentation) are divided into the following categories:

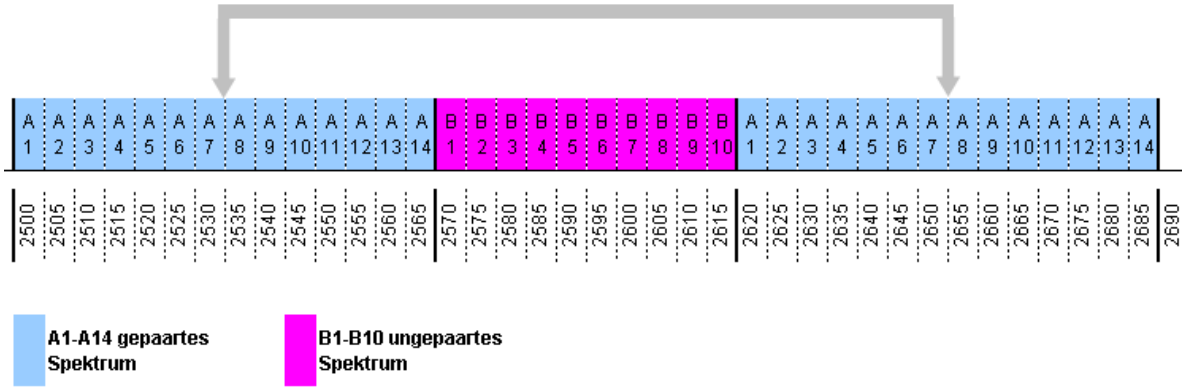
- Category A (2.6 GHz paired): 14 frequency blocks;
- Category B (2.6 GHz unpaired): 10 frequency blocks in total, 9 of which will be acquired in the principal stage (B10 will be assigned to the bidder who acquires B9).

Each frequency package is assigned a certain number of eligibility points (bidding points) which reflect the relative value of the frequencies. Table 1 below provides an overview of the frequency blocks, their respective designations in the auction procedure, and the number of eligibility points associated with each package. The position of each frequency block in the 2.6 GHz band is shown in Figure 1.

Table 1: Description of frequency blocks

Category	Frequencies	Package designation	Eligibility points
A: paired frequencies	2500 – 2505 MHz, paired with 2620 – 2625 MHz	A1	2*N for a bid on N blocks
	2505 – 2510 MHz, paired with 2625 – 2630 MHz	A2	
	2510 – 2515 MHz, paired with 2630 – 2635 MHz	A3	
	2515 – 2520 MHz, paired with 2635 – 2640 MHz	A4	
	2520 – 2525 MHz, paired with 2640 – 2645 MHz	A5	
	2525 – 2530 MHz, paired with 2645 – 2650 MHz	A6	
	2530 – 2535 MHz, paired with 2650 – 2655 MHz	A7	
	2535 – 2540 MHz, paired with 2655 – 2660 MHz	A8	
	2540 – 2545 MHz, paired with 2660 – 2665 MHz	A9	
	2545 – 2550 MHz, paired with 2665 – 2670 MHz	A10	
	2550 – 2555 MHz, paired with 2670 – 2675 MHz	A11	
	2555 – 2560 MHz, paired with 2675 – 2680 MHz	A12	
	2560 – 2565 MHz, paired with 2680 – 2685 MHz	A13	
	2565 – 2570 MHz, paired with 2685 – 2690 MHz	A14	
B: unpaired frequencies	2570 – 2575 MHz	B1	M-1 for a bid on M blocks
	2575 – 2580 MHz	B2	
	2580 – 2585 MHz	B3	
	2585 – 2590 MHz	B4	
	2590 – 2595 MHz	B5	
	2595 – 2600 MHz	B6	
	2600 – 2605 MHz	B7	
	2605 – 2610 MHz	B8	
	2610 – 2615 MHz	B9	
	2615 – 2620 MHz (not to be acquired separately; see explanations in Section 2.2 of the Tender Documentation)	B10	(not to be acquired separately)

Figure 1: Positions of frequency packages in the 2.6 GHz band



The paired frequency blocks are assigned 2*N eligibility points in the auction procedure, meaning that the activity of a participant who bids on N paired frequency blocks is equal to 2*N eligibility points. The unpaired frequency blocks are assigned M-1 eligibility points, meaning that the activity of a participant who bids on M unpaired frequency blocks is equal to M-1 eligibility points.

During the assignment stage, one additional block of unpaired spectrum (B10; 2615 – 2620 MHz) will be assigned to the bidder who acquires a package containing Frequency Block B9.

3.2.2. For the first round of the auction procedure as well as any subsequent rounds in which no bids have been placed on frequency blocks in a given category in any previous round, the current round price will be set equal to the reserve price for a frequency block in the respective category as shown in Table 2 below.

Table 2: Reserve prices for frequency blocks

Category	Reserve price in EUR
A	400,000
B	200,000

4. Principal stage

4.1. Bids and bidding

4.1.1. During the principal stage, bidders submit combinatorial package bids for abstract frequency blocks in the two categories (A and B). These bids are defined by three parameters: the bid amount, the number of frequency blocks in Category A and the number of frequency blocks in Category B the bidder would like to acquire at current round prices (NB: The number of blocks indicated may also be zero). A bid represents a binding offer on the part of the bidder to pay at most the specified price for a given combination of abstract frequency blocks.

- In the primary bidding stage (clock stage), bidders will place package bids for a certain number of abstract frequency blocks (which may also be zero) in each of the two categories at a bid amount which is calculated based on the prices set by the auctioneer for the current round.

- In the sealed-bid stage, bidders will submit package bids for a number of abstract frequency blocks in each of the two categories; bidders will be allowed to choose the number of blocks as well as the bid amount freely during this stage, but within the limits defined by the activity rules (cf. Rule 4.9).
- The auctioneer may begin the principal stage immediately with the sealed-bid stage. In such a case, the clock stage and the restrictions arising from Rule 4.9.6 will be eliminated entirely in accordance with Rule 4.9.8. The auctioneer will only do so if s/he is convinced that it is in the general interest of an efficient frequency assignment procedure.

4.1.2. Bids will be submitted electronically using an auction software program (cf. Section 6). The submission of bids by telephone will only be permitted in exceptional cases (e.g., technical difficulties). The auctioneer shall be responsible for deciding whether such an exception is warranted.

4.1.3. During the clock stage, the bid amounts themselves cannot be chosen freely. Instead, in each round, bidders are required to indicate the desired number of abstract frequency blocks (which may be zero) in each of the two categories, and the bid amount is calculated using the price set by the auctioneer for the current round (i.e., the number of blocks in a category is multiplied by the current round price for that category; the resulting amounts for each category are added up to yield the bid amount).

4.1.4. The current round price for an abstract frequency block in a category will be set by the auctioneer (cf. Rule 4.7). The software will calculate the bid amount automatically when the package bid is submitted during the clock stage.

4.1.5. During the sealed-bid stage, the bid amount can be chosen freely within the limits defined by the activity rules, and bids are to be submitted in accordance with Rule 4.9.

4.1.6. The amount of each bid submitted during the principal stage must comply with the rules regarding the bank guarantee as specified in Section 4.3.5 of the Tender Documentation. Bids which exceed the bidding limit arising from the respective bank guarantee will not be considered valid.

4.2. Eligibility

4.2.1. A bidder’s eligibility determines the maximum number of abstract frequency blocks on which that bidder can be active in any one round.

4.2.2. During the clock stage, a bidder is allowed to be active on any combination of abstract frequency blocks as long as the total eligibility points associated with the abstract frequency blocks contained in his/her package bid do not exceed that bidder's current eligibility and Rule 4.5 is not violated.

4.2.3. Each bidder's eligibility in the first round of the clock stage is determined on the basis of the information provided in the bidder's application (cf. Section 3.3 of the Tender Documentation). In subsequent rounds, the current eligibility of each bidder is determined by the activity rules.

4.2.4. During the sealed-bid stage, a bidder may be active on any combination of abstract frequency blocks as long as the total number of eligibility points associated with the package does not exceed that bidder's eligibility in the first round of the clock stage and the bid amounts adhere to the limits specified in the activity rules (cf. Rule 4.9).

4.3. Activity rules for the clock stage

4.3.1. The activity of a bidder in a given round is defined as the total number of eligibility points associated with the abstract frequency blocks in the bidder's package bid submitted in that round (cf. Rule 3.2.1). The eligibility in a given round is equal to the bidder's activity in the previous round.

4.3.2. Bidders who do not submit a combinatorial package bid (i.e., who submit a "zero bid") will be excluded from further participation in the clock stage. However, such bidders will still be allowed to submit sealed package bids (cf. Rule 4.9), and the bids submitted by such bidders during the clock stage will still be taken into account in the winner determination procedure (cf. Rule 4.10).

4.4. Round extensions during the clock stage

4.4.1. At the beginning of the auction, each bidder will be entitled to three round extensions.

4.4.2. These extensions provide a bidder with additional time to submit a bid during the clock stage. If a bidder who still has eligibility points as well as round extensions at his/her disposal does not submit a bid during the time allotted for a given round, then the software will automatically extend the round by a maximum of 30 minutes and subtract one of that bidder's round extensions. Round extensions will only be subtracted from those bidders who do not manage to submit a bid during the time allotted for a given round.

4.4.3. Bidders who do submit a bid during the time allotted for the round will not be allowed to revise their bids during the additional (extension) time. Bidders who have no more round extensions and do not manage to submit a bid during the time allotted for the round will not be allowed to submit a bid during an extension. In such cases, the auction software will automatically enter a zero bid on their behalf, and those bidders will be excluded from further participation in the clock stage (cf. Rule 4.3.2).

4.4.4. Rounds in which one or more bidders use a round extension will come to an end no later than either 30 minutes after the originally scheduled end time, or once all bidders who used a round extension during that round have successfully entered a bid, whichever is shorter.

4.4.5. No more than one round extension per bidder may be applied in each round.

4.5. Bidding rights

4.5.1. The (combinatorial) package bids submitted by any bidder who already holds frequencies in the 900 MHz or 1800 MHz band must not include more than six paired abstract frequency blocks (Category A) in each round (cf. Section 3.4 of the Tender Documentation).

4.5.2. The (combinatorial) package bids submitted by any other bidder must not include more than eight paired abstract frequency blocks (Category A) in each round.

4.5.3. In cases where a bidder's (combinatorial) package bid includes unpaired spectrum, then that bidder's package bid must include at least three unpaired abstract frequency blocks (Category B) and may include up to nine of those blocks.

4.6. Valid bids during the clock stage

4.6.1. A bid is only considered valid if it is submitted within the time period defined by the auctioneer for the given round or during the extended time allotted due to the exercise of a round extension. This also applies to the submission of bids by telephone.

4.7. Round price determination during the clock stage

4.7.1. In the first round of the clock stage, the price of abstract frequency blocks in each category will be set equal to the reserve price.

4.7.2. In subsequent rounds of the clock stage, the prices of blocks in each category will be determined as follows:

- The price of a paired abstract frequency block (Category A) will be increased if total demand for blocks in this category is greater than 14 abstract frequency blocks (i.e., if excess demand exists). If the total demand for this category is less than or equal to 14 abstract frequency blocks, the price will remain the same in the following round.
- The price of an unpaired abstract frequency block (Category B) will be increased if total demand for blocks in this category is greater than nine abstract frequency blocks (i.e., if excess demand exists). If the total demand for this category is less than or equal to nine abstract frequency blocks, the price will remain the same in the following round.

4.7.3. Prices will not be reduced; this means that the price of either category in any given round of the clock stage cannot be lower than the price of that category in the previous round.

4.7.4. The amount by which the round price is increased will be determined by the auctioneer. The price increase will generally be between 2% and 15%, but the auctioneer reserves the right to apply different price increments. All prices will be set in whole euro amounts.

4.8. End of the clock stage

4.8.1. The clock stage ends:

- automatically if no excess demand exists in either of the two categories; or
- by decision of the auctioneer, even if excess demand still exists in one or both categories.

The auctioneer will only end the clock stage if s/he is convinced that an immediate transition to the sealed-bid stage is in the general interest of an efficient frequency assignment procedure.

4.8.2. In both cases, the auction will proceed directly to the sealed-bid stage (as described below), and no further rounds will be held in the clock stage.

4.9. Activity rules and valid bids for the sealed-bid stage

4.9.1. A bid is only considered valid if it is submitted within the time period defined by the auctioneer for the sealed-bid stage. This also applies to the submission of bids by telephone.

4.9.2. The bids submitted in the sealed-bid stage must be in whole euro amounts.

4.9.3. Sealed supplementary bids can be submitted for:

- Combinations of abstract frequency blocks on which the bidder placed bids during the clock stage; and
- Combinations of abstract frequency blocks on which the bidder could have bid during the clock stage based on his/her eligibility (in the first round of the clock stage) and the rules set out in 4.5.

Only one sealed package bid can be submitted for any particular combination of abstract frequency blocks.

4.9.4. The minimum amount of a sealed combinatorial package bid will be either the sum of the reserve prices (cf. Rule 3.2.2) for the abstract frequency blocks included in the particular combination, or the highest clock bid submitted for that combination during the clock stage if the bidder submitted a bid for that package during the clock stage.

4.9.5. The following definitions are relevant for the determination of caps on bid amounts during the sealed-bid stage:

- "Zero bid": A bid of zero for a combination containing no frequency blocks.
- "Anchor round": The anchor round for a given Combination C is the round in which the bidder last had sufficient eligibility to bid on that combination of abstract frequency blocks.
- "Anchor combination": The anchor combination for a given Combination C is the package on which the bidder placed a bid instead of Combination C in the anchor round; the anchor combination is denoted as C'. C' can be a zero bid.
- "Anchor bid": The anchor bid for a given Combination C is the highest bid received for the anchor combination (i.e., for C'). The anchor bid is therefore either the supplementary bid submitted for the anchor combination during the sealed-bid stage or the highest clock bid submitted for the anchor combination (if no supplementary bid is placed on this combination).
- "Last clock bid": The last non-zero bid submitted in the clock stage.

4.9.6. The bid amounts for sealed combinatorial package bids are subject to caps according to the provisions below and the definitions in Rule 4.9.5:

- Bidders who were active in the last round of the clock stage can increase the bid amount for the package they submitted in the last clock round as desired. If a bidder placed his/her last clock bid in an earlier round, that bidder can submit a sealed supplementary bid on the same combination of frequency blocks, but the bid is capped at the value of that combination at the prices set in the round immediately following the round in which the bidder placed his/her last clock bid.
- The bid amount placed on any other combination of abstract frequency blocks is capped relative to its respective anchor bid.
- The bid amount for Combination C must not be higher than the corresponding anchor bid plus the price difference between Combination C and the corresponding Anchor Combination C' at the prices in the anchor round.

This means that bidders who remain active until the end of the clock stage can increase the caps for sealed supplementary bids (indirectly) by submitting a higher bid on the combination they bid on in the last clock round. Bidders who remain active in the last round of the clock stage may increase their bid for this combination by any amount. For combinations requiring fewer eligibility points than (or the same number as) the combination bid on in the last clock round, the last round of the clock stage will be considered the anchor round, and the combination of the last clock bid will be considered the anchor combination.

Bidders who do not remain active until the last round of the clock stage can increase the last clock bid up to the amount that they would have had to bid in the round when they first submitted a zero bid. All other sealed supplementary bids for combinations with more eligibility points than the combination of the last clock bid are limited relative to their respective anchor combinations. For combinations that are associated

with fewer eligibility points than (or the same number of eligibility points as) the combination of the last clock bid, the first round of the clock stage in which a zero bid was placed will be considered to be the anchor round, and the zero bid will be considered to be the anchor combination.

4.9.7. The auctioneer reserves the right to relax this restriction at the beginning of the sealed-bid stage for those supplementary bids where the anchor combination is not a zero bid. The limit will be relaxed by multiplying the value difference (between Combination C and Anchor Combination C') added to the anchor bid in accordance with Rule 4.9.6 by a factor of α (if the value difference is greater than zero) or $1/\alpha$ (if the value difference is less than zero). The auctioneer will only do so if s/he is convinced that it is in the general interest of an efficient frequency assignment procedure.

4.9.8. The auctioneer also reserves the right to suspend the limits arising from Rule 4.9.6 entirely, that is, to eliminate all upper limits on bid amounts for sealed package bids. The auctioneer will only suspend the limits arising from Rule 4.9.6 entirely if s/he is convinced that it is in the general interest of an efficient frequency assignment procedure.

The Appendix contains a number of examples that illustrate the application of the activity rules in detail.

4.10. Determination of winning bidders

4.10.1. After the end of the sealed-bid stage, the auctioneer will determine the combination of bids (among all bids submitted during the clock stage and the sealed-bid stage) which meets the following conditions:

- At most one combinatorial package bid per bidder is included.
- The total number of blocks acquired in each category must not be larger than the number of blocks available in that category.
- The sum of the bid amounts must not be lower than the sum of any other alternative combination of bids which meets the first two conditions.

4.10.2. If there is only one combination of combinatorial package bids that meets the conditions above, this will be considered the winning combination of bids.

4.10.3. If multiple combinations of combinatorial package bids meet the conditions set out in 4.10.1, then the combination of (combinatorial) package bids that is associated with the largest number of eligibility points will be considered to be the winning combination of bids.

4.10.4. If multiple combinations of combinatorial package bids meet the conditions defined under 4.10.1 and are associated with the same (largest) number of eligibility points, the combination of package bids that includes the largest number of winners will be considered the winning combination of bids.

4.10.5. If multiple combinations of combinatorial package bids meet the conditions defined under 4.10.1, are assigned the same (largest) number of eligibility points and include the same (largest) number of winners, the winning combination of bids will be determined by a process of random selection.

4.10.6. The winning bidders will be those whose bids are part of the winning combination of package bids. The winning bidders will receive the number of abstract frequency blocks in each category associated with their winning bids at the base price calculated according to Rule 4.11.

A description of the algorithm used to determine the winning bidders is provided in the Appendix.

4.11. Calculation of base prices

4.11.1. For each winning package bid, a base price will be calculated as the amount to be paid by the winning bidder for his/her winning bid. This means that base prices are not calculated for individual abstract frequency blocks in each category. Base prices will also be calculated jointly for all winning bidders.

4.11.2. The base price for each winning bid must be at least as high as the total of the reserve prices for the combination of abstract frequency blocks included in the winning bid. The base price for each winning package cannot be higher than the amount of the corresponding winning bid.

4.11.3. Base prices are the lowest overall prices which:

- meet the condition set out in Rule 4.11.2; and
- ensure that the winning combination of bids meets the conditions set out in Rule 4.10.1 at the respective base prices.

This means that base prices are the lowest prices the winning bidders would have had to bid (jointly) in order for their respective bids to be successful.

4.11.4. If there is only one combination of base prices that meets the conditions under 4.11.2 and 4.11.3, then those base prices (rounded up to the next whole euro amount) will be set as the base prices.

4.11.5. If there are multiple combinations of base prices that meet those conditions, then the combination of base prices (rounded up to the next whole euro amount) that is closest to the opportunity costs of each winning bid (calculated individually) will be defined as the relevant combination of base prices. The opportunity costs of each winning bid are calculated individually and refer to the lowest bid that complies with Rule 4.11.2 and that would ensure that the winning combination of bids meets the conditions of Rule 4.10.1 if all other winning bidders were to pay their original bid amount.

A description of the algorithm used to calculate base prices is provided in the Appendix.[1]

4.12. Information released to bidders

4.12.1. At the beginning of the clock stage (i.e., at the start of the first clock round), each bidder will be informed of his/her eligibility. This information will not be communicated to other bidders, nor will the auctioneer communicate the total number of eligibility points held by all bidders.

4.12.2. At the beginning of each round in the clock stage, the auctioneer will release at least the following information to each bidder:

- The current round;
- The time at which the round ends;

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[1] Due to numerical inaccuracies in the automated calculation of the base price, minimal deviations from the theoretical base price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated base price. See also the explanations on the calculation of base prices in the Appendix.

- The price of a frequency block in each category in the current clock round;
- The number of eligibility points currently held by the bidder;
- The remaining number of round extensions held by the bidder.

4.12.3. After the end of each round in the clock stage, the auctioneer will release the following information to each bidder:

- A summary of the combinatorial bid submitted by the bidder, the bidder's remaining number of eligibility points, and the bidder's remaining number of round extensions.

4.12.4. The auctioneer will inform each bidder accordingly once the clock stage has ended (cf. Rule 4.8).

4.12.5. At the beginning of the sealed-bid stage, the auctioneer will communicate at least the following information to each bidder:

- The time at which the sealed-bid stage ends;
- The degree to which bidding limits are to be relaxed (i.e., the value of parameter α in accordance with Rule 4.9.7).

4.12.6. After the end of the principal stage (i.e., at the end of the sealed-bid stage), the auctioneer will release the following information to each bidder:[2]

- The identity of each winning bidder in the principal stage; and
- The number of abstract frequency packages acquired by each winning bidder in each category.
- Each winning bidder will be informed of the base price for his/her winning bid. This information will not be communicated to other bidders.

5. Assignment stage

5.1. Assignment stage procedure

5.1.1. The purpose of the assignment stage is to determine how the available frequencies are distributed amongst the winning bidders of the principal stage, and to determine the price the bidders will ultimately pay for the frequencies they have won.

5.1.2. In this stage, two procedures will be carried out simultaneously:

- The assignment procedure for unpaired spectrum;
- The assignment procedure for paired spectrum.

The exact procedure for each category depends on the number of winning bidders and number of lots sold in that category.

5.1.3. If the number of blocks won in a category in the principal stage is lower than the number of available blocks (cf. Section 2.2 of the Tender Documentation), the frequency blocks will be assigned as follows:

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[2] Usually, the results should be available in less than one second. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results or additional processing capacity may be required. In such an improbable case, the submission of bids will be concluded and the results will be announced at a later point in time.

- Paired frequency blocks (Category A) will be assigned consecutively from 2500 MHz upward (paired with the corresponding frequency blocks from 2620 MHz upward); this means that any spectrum not acquired in this category will be in contiguous blocks at the upper end of the paired range.
- Unpaired frequency blocks (Category B) will be assigned consecutively from 2615 MHz downward; this means that any spectrum not acquired in this category will be in contiguous blocks at the lower end of the unpaired range.

5.1.4. If there is only one winning bid in a given category, then no assignment bid will be necessary for that category. The winning bidder will be assigned the relevant frequency blocks automatically in accordance with Rule 5.1.3.

5.1.5. If more than one bidder acquires abstract frequency blocks in a given category in the principal stage, the auctioneer will provide each winning bidder with a complete list of relevant assignment bid options for actual frequency blocks in that category. These options must meet the following conditions:

- The number of frequency blocks assigned in each category must be equal to the number of abstract frequency blocks the bidder has won in that category in the principal stage.
- Assignment options must not prevent the assignment of contiguous frequency blocks to other bidders who have won abstract frequency blocks in the respective category in the principal stage, or prevent the retention of contiguous unassigned spectrum at the upper end of the paired range or at the lower end of the unpaired range.

A description of the algorithm used to determine assignment bid options as well as examples can be found in the Appendix.[3]

5.2. Assignment bids and bidding

5.2.1. For each bidder, the auction software will determine all possible and relevant assignment options for the actual frequency blocks in each category; these options must meet the conditions defined under 5.1.2, 5.1.3 and 5.1.5. In the auction procedure, a bidder's assignment options will be identified uniquely by the far left (i.e., lowest) frequency block in the assignment option (i.e., the starting block).

5.2.2. In the assignment stage, the winning bidders from the principal stage will submit sealed bids for the assignment options defined for them in each category; these will be package bids for the actual frequency blocks included in an assignment option for a given bidder.

5.2.3. Bids will be submitted electronically using the auction software (cf. Section 6). The submission of bids by telephone will only be permitted in exceptional cases (e.g., technical difficulties). The auctioneer shall be responsible for deciding whether such an exception is warranted.

5.2.4. Bidders may freely choose the bid amounts for packages during the assignment stage. If a bidder does not submit a bid on a possible assignment option defined for

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[3] Usually, the assignment bid options should be available in less than one second. In certain situations (e.g., more than ten winning bidders in the paired range), this process may take significantly longer, or additional processing capacity may be required. In such an improbable case, delays may arise in the auction procedure.

him/her by the auction software, the auction software will automatically generate a zero bid for that assignment option.

5.3. Valid bids

5.3.1. During the assignment stage, a bid is only considered valid if it is submitted within the time period defined by the auctioneer for the assignment bidding round. This also applies to the submission of bids by telephone.

5.3.2. In the assignment stage, bids must be submitted in whole euro amounts. The minimum bid in the assignment stage is EUR 0. Bids are not subject to an upper limit.

5.3.3. For each assignment option on which a bidder does not bid, the auction software will automatically generate a bid with an amount of EUR 0 (i.e., a zero bid). If a bidder does not submit any bid within the time period defined by the auctioneer, the auction software will automatically place a zero bid for each assignment option in each category in which the bidder is allowed to bid.

5.4. Determination of winning bidders

5.4.1. After the end of the bidding round in the assignment stage, the auctioneer will determine the combination of assignment bids (among all assignment bids submitted by the bidders and generated automatically by the auction software) which meets the following conditions in each category (paired and unpaired spectrum):

- In each category, exactly one assignment bid per bidder must be included.
- The assignment of frequency blocks in connection with the assignment bids must ensure that each bidder receives contiguous blocks and that any unassigned frequency blocks are contiguous blocks at the upper end of the paired and the lower end of the unpaired frequency range (cf. Rules 5.1.2, 5.1.3 and 5.1.5).
- The sum of the assignment bid amounts must not be lower than the sum of any other alternative combination of assignment bids that meets the first two conditions.

5.4.2. If there is only one combination of assignment bids that meets the conditions above, this combination will be considered the winning combination of assignment bids.

5.4.3. If there are multiple combinations of assignment bids that meet the conditions under 5.4.1, a process of random selection will determine the winning combination of assignment bids.

5.4.4. Each bidder will be assigned the frequency blocks specified in the assignment bid which is included in the winning combination and will pay a top-up price determined in accordance with Rule 5.5.

5.5. Calculation of top-up prices

5.5.1. For each winning assignment bid, a top-up price will be calculated as the amount to be paid by each winning bidder from the principal stage for his/her winning assignment bid. Top-up prices will be calculated jointly for all bidders.

5.5.2. The top-up price for each winning assignment bid must be at least as high as the minimum bid for the respective assignment option (cf. Rule 5.3.2). The top-up price for each winning assignment bid cannot be higher than the amount of that bid.

5.5.3. Top-up prices are the lowest overall prices which:

- meet the condition set out in Rule 5.5.2; and

- ensure that the winning combination of bids meets the conditions specified in Rule 5.4.1 at the respective top-up prices.

This means that top-up prices are the lowest prices the winning bidders would have had to bid in order for their respective assignment bids to be successful.

5.5.4. If there is only one combination of top-up prices that meets the conditions under 5.5.2 and 5.5.3, then those prices (rounded up to the next whole euro amount) will be selected as the top-up prices.

5.5.5. If there are multiple combinations of top-up prices that meet those conditions, then the combination of top-up prices (rounded up to the next whole euro amount) that is closest to the opportunity costs of each winning assignment bid (calculated individually) will be selected as the relevant combination of top-up prices. The opportunity costs of each winning bid are calculated individually and refer to the lowest assignment bid that complies with Rule 5.5.2 and would ensure that the winning combination of assignment bids meets the conditions of Rule 5.4.1 if all other winning assignment bidders were to pay their original assignment bid amount.

A description of the algorithm used to determine top-up prices is provided in the Appendix.[4]

5.6. Overall price

5.6.1. The overall price to be paid by each winning bidder will be calculated as the sum of the base price (determined in the principal stage) and the top-up price (determined in the assignment stage).

5.7. Information released to bidders

5.7.1. Prior to the start of the assignment stage, the auctioneer will release at least the following information to each winning bidder from the principal stage:

- All relevant options for the assignment of actual frequency blocks;
- The time at which the assignment stage ends.

5.7.2. At the end of the auction (i.e., at the end of the assignment stage), the auctioneer will release at least the following information to each bidder:[5]

- The identity of each winning bidder;
- The frequency blocks to be assigned to each winning bidder in each category;
- The total price each winning bidder will have to pay for the frequency blocks, including an indication of the base price and top-up price.

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[4] Due to numerical inaccuracies in the automated calculation of the top-up price, minimal deviations from the theoretical top-up price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated top-up price. See also the explanations on the calculation of top-up prices in the Appendix.

[5] cf. footnote no. 2.

6. Auction software

6.1. General information

Bids will be submitted locally and electronically using the auction software. The submission of bids by telephone will only be permitted in exceptional cases (e.g., technical difficulties). The auctioneer shall be responsible for deciding whether such an exception is warranted. In such cases, the auctioneer will decide whether the current round is to be stopped and repeated, or whether the bidder experiencing technical difficulties should be allowed to submit bids without the auction software.

6.2. Procedure for auction rounds

From the bidders' perspective, each round will be divided into the following phases:

- Bid submission phase: Bids can only be placed during the bid submission phase. This phase will begin when the auctioneer announces the start of a round and when the bidding window is displayed. The bidding window will remain available until either the time allotted for the round (possibly including an extension) has passed or the bidder has sent bids to the auction server.
- Waiting for round results: The successful submission of bids will then be confirmed. The bidder must then wait until the auctioneer either closes or discontinues the round.
- Display of round results: If the auctioneer closes the round, the bids will then be evaluated and the information defined in the auction rules will be displayed to the bidders.
- Discontinuation and repetition of a round: If the auctioneer discontinues the round (i.e., does not close the round normally), the bids submitted will not be evaluated (i.e., the bids are discarded); instead, the round will be repeated.
- Waiting for the start of the next round: Once a round's results have been evaluated, the bidder must wait until the next round begins. As soon as the auctioneer sets the scheduled start time for the next round, the start time will be displayed to the bidders. The scheduled start time is the earliest time at which the auctioneer can open the next round; the actual start of the next round is triggered manually by the auctioneer and will thus be slightly later than the scheduled time. The time between the rounds will be at least **ten minutes**.

In cases where a bidder has no connection to the server or encounters technical difficulties with the bidding client, bids will be submitted as specified in Section 6.9.

6.3. Submission of bids

The bids themselves will be submitted in two steps:

- In the first step, the bidder enters one or more bids.
- In the second step, the bids are transmitted to the auction server.

The specific form in which bids are to be entered will depend on the current stage in the auction procedure:

- Clock stage: In this stage, the bidder must indicate how many abstract frequency blocks s/he would like to acquire in each category at the price defined for the given round.
- Sealed-bid stage: In this stage, the bidder may submit multiple supplementary package bids; in each package bid, the bidder must enter the bid amount and the number of blocks desired in each category.

- Assignment stage: In this stage, the bidders may submit bids on assignment options for actual frequency blocks. The possible assignment options will be determined by the auction software.

Once the bidder has entered the bids, the software will request that the bidder confirm those bids.

Once a bid has been confirmed, it is transmitted to the auction server and is thus considered binding. After that point, it is no longer possible to modify, supplement or withdraw a bid.

6.4. Definitive time

In principle, the time on the auction server's clock will be considered definitive for the auction procedure. The auction server will be synchronised using the Network Time Protocol (NTP).

The time information displayed in the bidding window will be synchronised with the time on the server clock at regular intervals. During the bid submission phase, the end time (according to the server's clock) for the round as well as the time remaining in the round will be displayed on the bidding client. For technical reasons (e.g., latency between the client and server), this time may deviate slightly from the actual time remaining (according to the server's clock). For this reason, the time remaining will be displayed only as "<1 min" during the last minute of a given round.

Note: The system time usually displayed in the task bar on the bidder's computer will not be synchronised and may therefore differ significantly from the server time.

6.5. Duration of rounds in the clock stage

The duration of auction rounds will be set by the auctioneer at the beginning of each round.

During the clock stage, each round will last between **20 and 60 minutes** (unless a round extension right is exercised).

Once a round begins, the end time of the round will be displayed in the bidding window.

The end time is that point in time by which the round will end if no bidder exercises a round extension.

In cases where an eligible bidder still has a round extension at his/her disposal, the extension will be exercised automatically if the bidder does not submit a bid (which may be a zero bid) by the end time for that round.

Once all eligible bidders have submitted a bid (including zero bids), the bid submission phase for that round will come to an end (even if the end time for the round has not yet been reached). After that time, the auctioneer can close the round. Therefore, a round may also be substantially shorter than the nominal duration defined by the auctioneer. This will make it possible to hold far more auction rounds per day than the nominal round duration would allow.

6.6. Duration of rounds in the sealed-bid stage and assignment stage

If the clock stage ends before 1:00 pm, the sealed-bid stage will begin no earlier than 9:00 am the next day. If the clock stage ends after 1:00 pm, the sealed-bid stage will begin no earlier than 1:00 pm the next day. The start time of the sealed-bid stage will be announced after the end of the clock stage.

The duration will be set by the auctioneer at the beginning of the round. Once the round begins, the end time of the round will be displayed in the bidding window.

The round will come to an end once all eligible bidders have submitted a bid or once the end time for the round has passed. No round extensions will be granted.

In the sealed-bid stage, the round duration will be between **2 and 4 hours**.

The start time of the assignment stage will be announced once the principal stage has been completed. The time period between the end of the principal stage and the start of the assignment stage will be at least 24 hours.

In the assignment stage, the round duration will be between **2 and 4 hours**.

6.7. Round extensions

During the clock stage, bidders may exercise round extensions (cf. the rules defined under 4.4). These extensions provide a bidder with additional time to submit a bid. If a bidder who still has eligibility points as well as round extensions at his/her disposal does not submit a bid during the time allotted for a given round, then the software will automatically extend the round by a **maximum of 30 minutes** and subtract one of that bidder's round extensions. Round extensions will only be subtracted from those bidders who do not manage to submit a bid during the time allotted for a given round.

Bidders who do submit a bid during the time allotted for the round will not be able to revise their bids during the additional (extension) time. Bidders who have no more round extensions and do not manage to submit a bid during the time allotted for the round will not be allowed to submit a bid during an extension. In such cases, the auction software will automatically enter a zero bid, and those bidders will be excluded from further participation in the clock stage (cf. Rule 4.3.2).

Rounds in which one or more bidders use a round extension will come to an end no later than either 30 minutes after the originally scheduled end time, or once all bidders who used a round extension during that round have successfully entered a bid, whichever is shorter.

No more than one round extension per bidder may be applied in each round.

6.8. Evaluation of bids and announcement of results

Once a round's bid submission phase ends, the auctioneer can trigger the evaluation of bids in that round. Once the results of this evaluation are available, the information defined in the auction rules will be displayed to the bidders. Until they are evaluated, the bids remain unknown (also to the auctioneer).

Usually, the results should be available in less than one second. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results, or additional processing capacity may be required. In such an improbable case, the submission of bids in the relevant round will be concluded and the results will be announced at a later point in time. This may lead to delays in the auction procedure.

6.9. Exception: Submission of bids without the bidding software

The submission of bids without the bidding software will only be permitted in exceptional cases (e.g., technical difficulties). In such cases, the auctioneer must be informed by telephone immediately. The same time limits apply to the submission of bids with and without the bidding software. Under no circumstances will it be possible to submit bids at a later time; bids received by telephone after the end time for the round has passed will therefore not be taken into account.

The submission of bids without the bidding software will be handled as follows:

1) The bidder is to inform the auctioneer about the exceptional circumstances.
2) The auctioneer will then decide whether the bids should be submitted without the bidding software (or the round should be discontinued and repeated).

3) The bidder will then convey his/her bid(s) by telephone to an RTR employee nominated by the auctioneer.
4) The bids will then be entered electronically and printed out, but not actually submitted.
5) The bids will then be sent to the bidder by fax.
6) The bidder must then sign the bid (or inform the auctioneer of any errors in the bid; in such cases, return to No. 3).
7) The bidder must then fax the signed bid to the auctioneer.
8) The bid will then be submitted by an RTR employee.

6.10. Discontinuation of auction rounds

The auctioneer may discontinue an ongoing round at any time and repeat the round at a later point in time.

The bids placed in that round will then be considered void and remain unknown to all parties (including the auctioneer).

6.11. Interruptions of the auction

The auctioneer may interrupt the auction at any time and continue it at a later point in time. In such cases, the bidders will be informed of the reason for the interruption.

6.12. Messages to bidders

The auctioneer will be able to send messages to bidders through the auction software. Bidders will be alerted of unread messages by means of a symbol on the lower right-hand side of the bidding window.

6.13. User manual

The regulatory authority plans to send out a user manual for the auction software along with the Rules of Procedure.

7. Organizational framework

Organizational details regarding the auction will be delivered to bidders in **July 2010**. This Web version covers only the main points with regard to organization.

7.1. Authorised bidders

Each applicant must provide the regulatory authority with the names of at least two and at most six representatives who are authorised to place legally binding bids on behalf of the company during the auction. This notification must be received in writing by **September 1, 2010** (cf. Annex A). The authorization to represent the applicant is to be verified by a current excerpt from the Commercial Register, or by submitting documents granting the corresponding power of attorney.

The security key required for the auction will only be delivered to an authorised bidder.

Note: For practical reasons, applicants are advised to announce their authorised bidders during the theoretical training session for bidders.

7.2. Location of the auction

The auctioneer will be located on the premises of the regulatory authority at Mariahilfer Strasse 77-79, A-1060 Vienna, Austria. It is left up to the bidders to decide on the location from which they submit their bids. However, this location must be announced to the regulatory authority in writing by **September 1, 2010**.

7.3. Computer used for bid submission (bidder's computer)

In order to submit bids, each bidder will require a PC with broadband Internet access (minimum downlink speed: 256 kBit/s; minimum uplink speed: 64 kBit/s).

As a minimum, the following software must be installed on the PC:

- Web browser: Current version of Mozilla Firefox or Microsoft Internet Explorer recommended;
- Oracle Java Standard Edition Version 6 (most recent Version 6 release);
- PDF viewer: Current version of Adobe Acrobat Reader recommended;
- Operating system:
    - Microsoft Windows XP, Vista or 7;
    - Current version of Linux Ubuntu;
    - Current version of Mac OS X.

Notes on the configuration of the bidder's computer: Functions which might trigger an automatic restart of the computer (e.g., automatic software distribution or software updates) should be deactivated during the auction in order to prevent unwanted interruptions of the bidding process.

The same applies to the computer's Internet connection: Connections which make use of an inactivity timeout or which are dropped after a maximum time period (e.g., forced disconnection after 8 hours) should be avoided due to potential interruptions of the bidding process.

It is possible – and entirely reasonable for the purpose of enhancing availability – for bidders to keep more than one computer available for the auction. For each bidder, however, only one computer may be connected to the auction server at a time. The same applies to the Internet connection; here it is also advisable to ensure that multiple connections – ideally based on different technologies – are available.

Should a computer malfunction, the bidder can switch to a different computer at any time and continue participating in the auction; however, the bidder will lose the time required to repeat the login process. Therefore, bidders are advised to submit bids quickly and not to wait until the last possible moment to submit their bids unless absolutely necessary.

7.4. Communication with bidders

For the purpose of additional communication between the bidder and the auctioneer, all bidders must provide the regulatory authority with a fax number, a telephone number and an e-mail address in writing by September 1, 2010.

All communication between the bidder and the auctioneer or auction software may be recorded by the auctioneer for the purpose of documenting the assignment procedure.

7.5. Training sessions for bidders

7.5.1. Theoretical training session for bidders

The regulatory authority plans to hold theoretical training sessions for bidders on RTR's premises in the period between July 27 and 29, 2010, and between August 30 and September 1, 2010. During this training session, the rules of the auction will be explained, and the bidders will be familiarised with the auction software. An individual appointment will be made for each bidder, and the regulatory authority will make efforts to account for scheduling requests from the bidders. Training session appointments will be coordinated immediately after the end of the tender submission period; however, sessions will only be held with those applicants who are actually admitted to the auction. The regulatory authority plans to decide on admission to the auction on July 26, 2010.

7.5.2. Practical training session for bidders

The regulatory authority plans to hold a practical training session for bidders in the period between September 7 and 10, 2010. The practical session will enable bidders to practice placing bids from their own premises in the course of a test auction. An individual appointment will be made for each bidder, and the regulatory authority will make efforts to account for scheduling requests from the bidders.

7.6. Test auction

A brief test auction will be carried out in the morning on the day of the actual auction. This will be done in order to test the overall system with all of the bidders. This test auction will be discontinued after just a few rounds. During the test auction, bidders will use the same login data as in the real auction.

7.7. Auction

Details regarding the auction will be delivered to bidders in July 2010.

7.8. Simulation tool

Once the tender submission period has ended, the regulatory authority will provide each applicant with access to a simulation tool for the purpose of verifying the procedures for determining winning bids and prices. For this purpose, applicants will be able to send an Excel file containing bids to a Web-based interface. The simulation tool will then determine the winning bidders and the respective winning combinations and prices to be paid.

Applicants will be provided with a description as well as a sample file for the simulation tool.

This tool will be made available from July 12, 2010 (4:00 pm) to September 10, 2010 (3:00 pm) at the latest.

Appendix A: Examples of maximum bids in the sealed-bid stage

This section provides two examples of the application of the activity rules defined in Section 4.9 for the sealed-bid stage.

Example 1

The bidder applies for eight eligibility points. In the first ten rounds of bidding, the bidder places bids on four paired blocks. In Round 11, the bidder places a bid on only three paired blocks and continues to bid on three paired blocks until the end of the clock stage.

The prices in Round 11 are as follows:

- EUR 2.5 million for a paired block;
- EUR 1 million for an unpaired block.

The prices in the last round of the clock stage are as follows:

- EUR 3 million for a paired block;
- EUR 1.2 million for an unpaired block.

The bidder's last package bid in the clock stage amounts to EUR 9 million for three paired blocks. The bidder can choose to increase this bid by any amount, that is, to submit a sealed package bid for three paired blocks at the amount of EUR 9 million + EUR X (where X is the amount by which the last clock bid is increased). All other package bids are capped relative to this bid.

In this example, the package of three paired blocks (six eligibility points) is the anchor combination for all supplementary bids.

The bid amounts for all larger packages (i.e., those associated with seven or eight eligibility points) are capped by the anchor bid of EUR 9 million + EUR X and the prices in Round 11 (= anchor round for larger packages).

The bid amounts for all packages associated with six eligibility points or less are capped by the anchor bid and the prices in the last clock round.

Given that a bid for unpaired spectrum must include at least three frequency blocks, the bidder can submit the following sealed bids (with a maximum of eight eligibility points):

Table 3: Relative caps in the sealed-bid stage – Example 1

Package		Eligibility points	Relative caps
Blocks in Category A	Blocks in Category B		
1	0	2	EUR 9 million + EUR X - 2 x EUR 3 million = EUR 3 million + EUR X
0	3	2	EUR 9 million + EUR X - 3 x EUR 3 million + 3 x EUR 1.2 million = EUR 3.6 million + EUR X
1	3	4	EUR 9 million + EUR X - 2 x EUR 3 million + 3 x EUR 1.2 million = EUR 6.6 million + EUR X
2	0	4	EUR 9 million + EUR X - 1 x EUR 3 million = EUR 6 million + EUR X
1	4	5	EUR 9 million + EUR X - 2 x EUR 3 million + 4 x EUR 1.2 million = EUR 7.8 million + EUR X
1	5	6	EUR 9 million + EUR X - 2 x EUR 3 million + 5 x EUR 1.2 million = EUR 9 million + EUR X
2	3	6	EUR 9 million + EUR X - 1 x EUR 3 million + 3 x EUR 1.2 million = EUR 9.6 million + EUR X
0	8	7	EUR 9 million + EUR X - 3 x EUR 2.5 million + 8 x EUR 1 million = EUR 9.5 million + EUR X
1	6	7	EUR 9 million + EUR X - 2 x EUR 2.5 million + 6 x EUR 1 million = EUR 10 million + EUR X
2	4	7	EUR 9 million + EUR X - 1 x EUR 2.5 million + 4 x EUR 1 million = EUR 10.5 million + EUR X
0	9	8	EUR 9 million + EUR X - 3 x EUR 2.5 million + 9 x EUR 1 million = EUR 10.5 million + EUR X
1	7	8	EUR 9 million + EUR X - 2 x EUR 2.5 million + 7 x EUR 1 million = EUR 11 million + EUR X
2	5	8	EUR 9 million + EUR X - 1 x EUR 2.5 million + 5 x EUR 1 million = EUR 11.5 million + EUR X
3	3	8	EUR 9 million + EUR X + 3 x EUR 1 million = EUR 12 million + EUR X
4	0	8	EUR 9 million + EUR X + 1 x EUR 2.5 million = EUR 11.5 million + EUR X

If the auctioneer relaxes the caps in accordance with Rule 4.9.7 and sets α = 2, the bidder may submit the following sealed bids, among others (partial list):

Table 4: Relative caps in the sealed-bid stage – Example 1 (α=2)

Package		Eligibility points	Relative caps
Blocks in Category A	Blocks in Category B		
1	0	2	EUR 9 million + EUR X - 0.5 x 2 x EUR 3 million = 6 million + EUR X
1	3	4	EUR 9 million + EUR X + 0.5 x (3 x EUR 1.2 million - 2 x EUR 3 million) = EUR 7.8 million + EUR X
2	0	4	EUR 9 million + EUR X - 0.5 x 1 x EUR 3 million = EUR 7.5 million + EUR X
1	4	5	EUR 9 million + EUR X + 0.5 x (4 x EUR 1.2 million - 2 x EUR 3 million) = EUR 8.4 million + EUR X
1	5	6	EUR 9 million + EUR X + 2 x (5 x EUR 1.2 million - 2 x EUR 3 million) = EUR 9 million + EUR X
2	5	8	EUR 9 million + EUR X + 2 x (5 x EUR 1 million - 1 x EUR 2.5 million) = EUR 14 million + EUR X
3	3	8	EUR 9 million + EUR X + 2 x 3 x EUR 1 million = EUR 15 million + EUR X
4	0	8	EUR 9 million + EUR X + 2 x 1 x EUR 2.5 million = EUR 14 million + EUR X

Example 2

Let us assume that Bidder A has only submitted combinatorial package bids for paired blocks. This bidder begins the auction with 12 eligibility points, and s/he bids on six paired blocks in the first round. The reserve prices for both categories are shown in Table 5. Therefore, Bidder A places a bid of EUR 2.4 million for six paired blocks in the first round.

Table 5: Reserve prices for frequency blocks (Example 2)

Category	Reserve price in EUR
A	400,000
B	200,000

Bidder A continues to bid on six paired blocks until s/he reduces his/her bid to five paired blocks in Round 5. The price of a paired block in round 5 is EUR 800,000. The package of five paired blocks is therefore Bidder A's anchor combination for all packages requiring more than ten eligibility points (i.e., for all combinations with 11 and 12 eligibility points). The anchor round for those larger packages is Round 5.

In the following rounds, Bidder A continues to bid on five paired blocks, but eventually reduces his/her bid to four paired blocks in Round 10. The price of a paired block in round 10 is EUR 1.2 million. The package of four paired blocks is therefore Bidder A's anchor combination for all packages requiring more than eight but less than 11 eligibility points (packages worth 11 or more eligibility points are linked to the anchor bid for five paired blocks).

Bidder A then continues to bid on four paired blocks until the end of the clock stage. In the last round of the clock stage, Bidder A bids on four paired blocks, and the price of one paired block in that round is EUR 2.4 million. Therefore, Bidder A places a combinatorial package bid of EUR 9.6 million for four paired blocks in the last round. The package of four paired blocks is thus the relevant anchor combination for all package bids requiring eight eligibility points or less.

In the sealed-bid stage, Bidder A can submit an unlimited bid for a package of four paired blocks, as that was the bidder's last combinatorial package bid during the clock stage and the bidder remained active until the end of that stage.

Let us assume that the bidder places a sealed bid of EUR 16 million for a package of four paired blocks. This is Bidder A's anchor bid for the anchor combination of four paired blocks; this anchor bid limits the maximum bid amounts for all supplementary bids for packages with associated eligibility of less than 11 eligibility points.

In addition, assume that Bidder A would also like to submit the following sealed combinatorial package bids:

A ***bid for three paired blocks:*** This package is associated with six eligibility points, and the corresponding cap is thus linked to the anchor bid for a package of four paired blocks and the value difference in the anchor round (i.e., the last round in the clock stage in which the bidder was eligible to bid for three paired blocks, according to Rule 4.9.5). Specifically, the cap for this package is calculated as follows: The price of a paired block in the last round of the clock stage (EUR 2.4 million) is subtracted from the relevant anchor bid of EUR 16 million (the value difference between 3 and 4 paired blocks is EUR 2.4 million). This means that the bidder's maximum bid for a package with three paired blocks is EUR 13.6 million.

A ***bid for five paired blocks:*** This package is associated with ten eligibility points, and the bidder was last eligible to bid on such a package in Round 10. Therefore, Round 10 is the anchor round for the supplementary bid on a package of five paired blocks. However, as Bidder A placed a bid for a package of four paired blocks in that round, his/her eligibility was reduced to 8 eligibility points in Round 11 (cf. Rule 4.3.1). The relevant anchor bid for a package of five paired blocks is thus also the highest bid for four paired frequency blocks, that is, the supplementary bid of EUR 16 million submitted by the bidder. At a price of EUR 1.2 million for one paired block in the anchor round (Round 10), the cap for a package of five paired blocks is calculated as the sum of the anchor bid (EUR 16 million) *plus* the value difference between the package of five paired blocks and the package of four paired blocks at the price in the relevant anchor round (Round 10: EUR 1.2 million), that is, EUR 17.2 million. Assume that the bidder places a sealed bid of EUR 17 million on this combination. This means that the anchor bid for the anchor combination of five paired blocks – on the basis of which the maximum bid amounts for all larger packages are determined – is not equal to the last clock bid for that combination (five blocks at the price defined in Round 9), but to the sealed supplementary bid of EUR 17 million.[6]

A ***bid for six paired blocks:*** This package is associated with 12 eligibility points, and the bidder was last eligible to bid on such a package in Round 5, which is therefore the anchor round. However, in that round, the bidder submitted a bid for five paired blocks, and this is

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[6] If the bidder had chosen not to submit a sealed supplementary bid for the combination of five paired blocks, then all subsequent supplementary bids on larger packages would have been limited relative to the last clock bid for the combination of five paired blocks. The last clock bid for that combination was submitted in Round 9 at a block price of less than EUR 1.2 million, meaning that the anchor bid for packages requiring more than ten eligibility points would not be EUR 17 million, but less than EUR 6 million.

thus the relevant anchor combination for packages associated with more than ten eligibility points. The relevant anchor bid for this combination (in this case the sealed supplementary bid of EUR 17 million for five paired blocks) and the price of EUR 800,000 for one paired block in the relevant anchor round (Round 5) yield a maximum bid amount of EUR 17.8 million for a supplementary bid for six paired blocks (cf. Rule 4.9.6).

A ***bid for nine unpaired blocks:*** This package is associated with eight eligibility points (nine minus one), and the bidder was last eligible to bid on such a package in the last round of the clock stage. The last clock round is therefore the relevant anchor round. The relevant anchor combination is a combination of four paired blocks, and the relevant anchor bid is Bidder A's supplementary bid of EUR 16 million for that combination. The maximum bid for this package is calculated using the anchor bid (EUR 16 million) and the value difference between this combination and the anchor combination at the price in the relevant anchor round (i.e., the last clock round). This value difference is calculated by subtracting the value of the anchor combination (4 x EUR 2.4 million = EUR 9.6 million) from the value of the combination desired by the bidder (9 x EUR 1 million = EUR 9 million). Therefore, the maximum bid is EUR 16 million minus EUR 0.6 million, which comes to EUR 15.4 million.

Table 7 below lists the maximum bids for all possible sealed combinatorial package bids for Bidder A. The calculations are based on the assumption that Bidder A submits (as described above) a sealed combinatorial package bid of EUR 17 million for five paired blocks as well as a sealed combinatorial package bid of EUR 17.5 million for six paired blocks. The prices of blocks in each relevant round are shown in Table 6.

Table 6: Relevant round prices – Example 2

Round	Price of one paired block	Price of one unpaired block
5	EUR 800,000	EUR 200,000
10	EUR 1.2 million	EUR 500,000
Last round of clock stage	EUR 2.4 million	EUR 1 million

Table 7: Maximum bids in the sealed-bid stage – Example 2

Package	Eligibility points	Maximum bid

Blocks in Category A	Blocks in Category B		
0	3	2	EUR 16 million - 4 x EUR 2.4 million + 3 x EUR 1 million = EUR 9.4 million
0	4	3	EUR 16 million - 4 x EUR 2.4 million + 4 x EUR 1 million = EUR 10.4 million
0	5	4	EUR 16 million - 4 x EUR 2.4 million + 5 x EUR 1 million = EUR 11.4 million
0	6	5	EUR 16 million - 4 x EUR 2.4 million + 6 x EUR 1 million = EUR 12.4 million
0	7	6	EUR 16 million - 4 x EUR 2.4 million + 7 x EUR 1 million = EUR 13.4 million
0	8	7	EUR 16 million - 4 x EUR 2.4 million + 8 x EUR 1 million = EUR 14.4 million
0	9	8	EUR 16 million - 4 x EUR 2.4 million + 9 x EUR 1 million = EUR 15.4 million
1	0	2	EUR 16 million - 3 x EUR 2.4 million = EUR 8.8 million
1	3	4	EUR 16 million - 3 x EUR 2.4 million + 3 x EUR 1 million = EUR 11.8 million
1	4	5	EUR 16 million - 3 x EUR 2.4 million + 4 x EUR 1 million = EUR 12.8 million
1	5	6	EUR 16 million - 3 x EUR 2.4 million + 5 x EUR 1 million = EUR 13.8 million
1	6	7	EUR 16 million - 3 x EUR 2.4 million + 6 x EUR 1 million = EUR 14.8 million
1	7	8	EUR 16 million - 3 x EUR 2.4 million + 7 x EUR 1 million = EUR 15.8 million
1	8	9	EUR 16 million - 3 x EUR 1.2 million + 8 x EUR 0.5 million = EUR 16.4 million
1	9	10	EUR 16 million - 3 x EUR 1.2 million + 9 x EUR 0.5 million = EUR 16.9 million
2	0	4	EUR 16 million - 2 x EUR 2.4 million = EUR 11.2 million
2	3	6	EUR 16 million - 2 x EUR 2.4 million + 3 x EUR 1 million = EUR 14.2 million

2	4	7	EUR 16 million - 2 x EUR 2.4 million + 4 x EUR 1 million = EUR 15.2 million
2	5	8	EUR 16 million - 2 x EUR 2.4 million + 5 x EUR 1 million = EUR 16.2 million
2	6	9	EUR 16 million - 2 x EUR 1.2 million + 6 x EUR 0.5 million = EUR 16.6 million
2	7	10	EUR 16 million - 2 x EUR 1.2 million + 7 x EUR 0.5 million = EUR 17.1 million
2	8	11	EUR 17 million - 3 x EUR 0.8 million + 8 x EUR 0.2 million = EUR 16.2 million
2	9	12	EUR 17 million - 3 x EUR 0.8 million + 9 x EUR 0.2 million = EUR 16.4 million
3	0	6	EUR 16 million - EUR 2.4 million = EUR 13.6 million
3	3	8	EUR 16 million - EUR 2.4 million + 3 x EUR 1 million = EUR 16.6 million
3	4	9	EUR 16 million - EUR 1.2 million + 4 x EUR 0.5 million = EUR 16.8 million
3	5	10	EUR 16 million - EUR 1.2 million + 5 x EUR 0.5 million = EUR 17.3 million
3	6	11	EUR 17 million -2 x EUR 0.8 million + 6 x EUR 0.2 million = EUR 16.6 million
3	7	12	EUR 17 million -2 x EUR 0.8 million + 7 x EUR 0.2 million = EUR 16.8 million
4	0	8	Unlimited; bid submitted by Bidder A: EUR 16 million
4	3	10	EUR 16 million + 3 x EUR 0.5 million = EUR 17.5 million
4	4	11	EUR 17 million - EUR 0.8 million + 4 x 0.2 million = EUR 17 million
4	5	12	EUR 17 million - EUR 0.8 million + 5 x EUR 0.2 million = EUR 17.2 million
5	0	10	EUR 16 million + EUR 1.2 million = EUR 17.2 million; bid submitted by Bidder A: EUR 17 million
5	3	12	EUR 17 million + 3 x EUR 0.2 million = EUR 17.6 million
6	0	12	EUR 17 million + EUR 0.8 million = EUR 17.8 million

Appendix B: Calculation of base prices

Description of the algorithm used to calculate base prices in accordance with Rule 4.11.

The procedure described below generates base prices which meet the conditions specified in Rule 4.11.

Let N be the set of bidders participating in the auction. The winner determination procedure in accordance with Rule 4.10 specifies a set of winning bidders W, and the sum of the winning bids is denoted as v. v^{-C} denotes the sum of winning bids identified in the winner determination procedure if the bidders in $C \subseteq N$ did not take part in the auction, and $\sigma(C) = (v - v^{-C})$ refers to the decrease in the total winning bid amount if the bidders in $C \subseteq N$ did not take part in the auction.

a) For each winning bidder $j \in W$, calculate v^{-j} as the sum of the bid amounts which would result from the winner determination procedure if Bidder j had not submitted a bid.

b) For each winning bidder $j \in W$, determine the minimum bid m_j relevant to the package of the winning bid (cf. Rule 3.2.2) and the maximum discounts $d_j^{\max} = \min[b_j - m_j, \sigma(\{j\})]$ where b_j is the bid amount associated with the winning bid of Bidder j.

c) Generate a list of constraints B for a linear program and initialise it as $B = \left\{ \forall j \in W : d_j \leq d_j^{\max}, \sum_{j=1}^{J} d_j \leq \sigma(W) \right\}$.

d) Solve the following maximisation problem: $\max \sum_{j \in W} d_j$ subject to the constraints in B and the constraint that the individual values of d_j cannot be negative. In general, many solutions to this maximisation problem may exist. If this is the case, pick a random solution d^*.

e) Reduce the bid amounts for all bids submitted by Bidder j by d_j^*; if this yields a negative bid amount, set this bid amount to zero.

f) Determine the winning bids in accordance with Rule 4.10 using the modified bid amounts and calculate the corresponding total winning bid amount.

g) If the original winning bids (at the reduced bid amounts) do not yield a lower value than the total winning bid amount calculated in Step f), then the maximum discount which can be granted to all winning bidders is $D^* = \sum_{j \in W} d_j^*$. Proceed to Step i).

h) If there are winning bidders from the original winner determination procedure who are no longer among the winning bidders given the modified bids, then add the following constraint to the list of constraints in B:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders who are no longer among the winners using the modified bid amounts. Proceed to Step d).

i) Solve the following minimisation problem: $\min \sum_{j \in W} \left(d_j - \sigma(\{j\}) \right)^2$ subject to the constraints in B, the constraint that the individual values of d_j cannot be negative,

and the constraint that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts, i.e., $\sum_{j \in W} d_j = D^*$. Let d^{**} be the solution to this minimisation problem.

j) Reduce the bid amounts for all bids submitted by Bidder j by d_j^{**}; if this yields a negative bid amount, set this bid amount to zero.

k) Determine the winning bids in accordance with Rule 4.10 using the modified bid amounts and calculate the corresponding total winning bid amount.

l) If the original winning bids (at the modified bid amounts) do not yield a lower value than the total winning bid amount calculated in Step k), then the values of d_j^{**} are the relevant discounts and the base prices are equal to the winning bids reduced by those amounts, that is, $p_j = b_j - d_j^{**}$ for all $j \in W$.

m) If there are winning bidders from the original winner determination procedure who are no longer among the winning bidders given the modified bids, then add the following constraint to the list of constraints in B:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders who are no longer among the winners using the modified bid amounts. Proceed to Step i).

Due to numerical inaccuracies in the automated calculation of base prices (minimum revenue core prices), minimal deviations from the theoretical base price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated base price. In very rare cases, the base price calculated may be a few euros higher than the theoretical base price.

Usually, the results should be available in less than one second. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results, or additional processing capacity may be required. In such an improbable case, the submission of bids will be concluded and the results will be announced at a later point in time.

Example 3: Calculation of base prices

In this example, six bidders (Alan, Bob, Carl, Doris, Emma and Fred) bid on 14 paired and nine unpaired frequency blocks in a combinatorial clock auction. During the principal stage, the bids listed in Table 8 are submitted. The minimum bids are the same as in the examples above (cf. Table 5).

Table 8: Combinatorial package bids in the principal stage – Example 3

Bidder	Package		Package bid (principal stage)
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	5	0	EUR 14.8 million
	4	0	EUR 14 million
Bob	6	4	EUR 21.8 million
	6	3	EUR 20.2 million
	5	4	EUR 20 million
	5	3	EUR 19.2 million
Carl	4	0	EUR 16 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 9.4 million
	0	5	EUR 9 million

The package bids highlighted in blue are the winning bids because they (clearly) represent the combination which maximises the revenue from the auction subject to it being able to be accommodated in the available spectrum (14 paired and nine unpaired abstract frequency blocks). The total revenues from this combination amount to EUR 60.8 million. The bidders who were not successful and their respective bids are highlighted in gray.

The description below follows the algorithm discussed in this Appendix.

Steps a) and b): For each winning bidder, calculate the sum of the bid amounts which would result from the winner determination procedure if that bidder had not submitted a bid. Determine the minimum bids and the maximum discounts.

Thus, the first step toward finding the base prices is to calculate the maximum discounts for each of the winning bidders.

If Alan had not taken part in the auction, the four paired abstract frequency blocks which he won could have not been awarded, as none of the unsuccessful bidders placed a combinatorial package bid for four (or fewer) paired abstract frequency blocks. This means that the opportunity cost for Alan's winning bid is zero. As base prices are subject to the condition that they must be at least as high as the reserve prices for this package (cf. Rule 4.11.3), the base price for Alan's winning bid is at least EUR 1.6 million.

Bob acquired a total of six paired and four unpaired abstract frequency blocks. If Bob had not participated in the auction, Alan would have won an additional paired abstract frequency block (thus acquiring a total of five), and Doris would have been able to acquire the four unpaired abstract frequency blocks. As a result, there would have been another winning bidder (Doris) in addition to the existing winners (Alan, Carl and Fred). The next best combination in terms of revenue would achieve EUR 46.8 million. This means that the maximum discount is EUR 14 million, and the opportunity cost of Bob's bid amounts to EUR 7.8 million.

If Carl had not taken part in the auction, the next best result in terms of revenue maximisation would have been to assign five paired abstract frequency blocks to Alan (with a package bid of EUR 14.8 million). The total bid amount would have thus fallen to EUR 45.6 million,

meaning that the maximum discount that might be granted to Carl is EUR 15.2 million. Subtracting this value from his bid yields the opportunity cost of his winning bid which equals EUR 0.8 million. As this value is lower than the sum of reserve prices for his winning package, the potential base price for Carl's winning bid is EUR 1.6 million.

Fred won five unpaired abstract frequency blocks. If Fred had not participated in the auction, then Emma would have been able to acquire five unpaired abstract frequency blocks. Therefore, the next best maximum revenue amount if Fred had not taken part in the auction would be EUR 59.8 million, which yields a maximum discount of EUR 1 million for Fred; the opportunity cost of Fred's winning bid is hence EUR 8 million.

The results of the calculation of the maximum discounts (Step b)) and the resulting potential base prices are shown in Table 9.

Table 9: Potential base prices – Example 3

Bidder	Next best maximum revenue amount	Maximum discount	Potential base price
Alan	EUR 46.8 million	Minimum of EUR 60.8 million - EUR 46.8 million = EUR 14.0 million and EUR 14 million - EUR 1.6 million = EUR 12.4 million i.e., EUR 12.4 million	EUR 1.6 million
Bob	EUR 46.8 million	Minimum of EUR 60.8 million - EUR 46.8 million = EUR 14 million and EUR 21.8 million - EUR 3.2 million = EUR 18.6 million i.e., EUR 14 million	EUR 7.8 million
Carl	EUR 45.6 million	Minimum of EUR 60.8 million - EUR 45.6 million = EUR 15.2 million and EUR 16 million - EUR 1.6 million = EUR 14.4 million i.e., EUR 14.4 million	EUR 1.6 million
Fred	EUR 59.8 million	Minimum of EUR 60.8 million - EUR 59.8 million = EUR 1 million and EUR 9 million - EUR 1 million = EUR 8 million i.e., EUR 1 million	EUR 8 million

Step c): We then generate a list of constraints and initialise it with the conditions that the individual discounts must not be higher than the maximum discounts calculated in the previous step, and that the overall sum of discounts over all bidders together must not be larger than the reduction in the total bid amount if the bidders had not taken part in the auction.

If none of the winning bidders had participated in the auction, Emma and Doris would have won and the total bid amount would have amounted to EUR 15 million. This means that the list of constraints includes the following conditions:

$$d_{Alan} \geq 0$$
$$d_{Bob} \geq 0$$
$$d_{Carl} \geq 0$$
$$d_{Fred} \geq 0$$
$$d_{Alan} \leq 12.4\,Mio$$
$$d_{Bob} \leq 14\,Mio$$
$$d_{Carl} \leq 14.4\,Mio$$
$$d_{Fred} \leq 1\,Mio$$
$$d_{\{Alan,Bob,Carl,Fred\}} \leq 45.8\,Mio$$

Step d): Maximise the sum of the individual discounts subject to these constraints.

The solution to this maximisation problem given the constraints is trivial:

$$d^*_{Alan} = 12.4\,Mio$$
$$d^*_{Bob} = 14\,Mio$$
$$d^*_{Carl} = 14.4\,Mio$$
$$d^*_{Fred} = 1\,Mio$$

Steps e), f) and g): Reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero. Then determine the winning bids in accordance with Rule 4.10 using the modified bid amounts and calculate the corresponding total winning bid amount. If the original winning bids do not yield a lower value given the reduced bid amounts, proceed to Step i).

If we reduce all of the winning bidders' package bids by the calculated maximum discounts and then calculate the resulting maximum total bid amount on the basis of those modified bid amounts, it becomes clear that the latter amount is no higher than the sum of the original winning bids less the corresponding discounts. The combination of the original winning bids therefore remains a potential winning combination (although it may be in a tie with other combinations).[7] This means that the potential base prices shown in Table 9 represent the actual base prices, as illustrated in Table 10.

Table 10: Modified package bids – Example 3

Bidder	**Package**		**Modified bid (after application of discount)**
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	5	0	EUR 2.4 million
	4	0	EUR 1.6 million
Bob	6	4	EUR 7.8 million
	6	3	EUR 6.2 million
	5	4	EUR 6 million

[7] In this case, the sum of the modified winning bids equals EUR 19 million. The same total bid amount would result if Alan's first bid, Carl's and Doris' bids as well as Fred's second bid were accepted.

	5	3	EUR 5.2 million
Carl	4	0	EUR 1.6 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 8.4 million
	0	5	EUR 8 million

Steps i), j), k) and l): Minimise the sum of squared differences between the individual discounts and the individual maximum discounts subject to the constraints and the condition that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts. Reduce the bid amounts for all of the winners' bids by the discount calculated in this way; if this yields a negative bid amount, set this bid amount to zero. Determine the winning bids in accordance with Rule 4.10 using the modified bid amounts and calculate the corresponding total winning bid amount. If the original winning bids (at the reduced bid amounts) do not yield a lower value than the total bid amount calculated in Step k), then the discounts found are the relevant discounts and the base prices are equal to the winning bids reduced by those amounts.

The quadratic optimisation yields the discounts found at the end of the linear optimisation. Those discounts do not change the result of the winner determination procedure and are thus the discounts with which the corresponding base prices are calculated. The base prices are shown in Table 11.

Table 11: Base prices for winning bids – Example 3

Bidder	Base price
Alan	EUR 1.6 million
Bob	EUR 7.8 million
Carl	EUR 1.6 million
Fred	EUR 8 million
Total:	EUR 19 million

Example 4: Calculation of base prices – Minimisation of total value

Base prices are not generally identical to opportunity costs, as is the case in Example 3 (with the exception of minimum bid restriction for Alan and Carl); in principle, it cannot be ensured that reducing the winning bids by the maximum discounts will leave the winning bids unchanged. Under these circumstances, the base prices need to be higher than the individual opportunity costs of winning bids.

Let us assume that an additional bidder, Greg, takes part in the auction, and hence a total of seven bidders (Alan, Bob, Carl, Doris, Emma, Fred and Greg) bid on paired and unpaired abstract frequency blocks in a combinatorial clock auction. During the principal stage, the bids listed in Table 12 are submitted.

Alan, Bob, Carl, Doris, Emma and Fred submit the same bids as in the previous example. Greg places a combinatorial package bid for four paired abstract frequency blocks and five unpaired abstract frequency blocks. The winning combination of bids is again highlighted in blue below.

Table 12: Combinatorial package bids in the principal stage – Example 4

Bidder	Package		Package bid (principal stage)
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	5	0	EUR 14.8 million
	4	0	EUR 14 million
Bob	6	4	EUR 21.8 million
	6	3	EUR 20.2 million
	5	4	EUR 20 million
	5	3	EUR 19.2 million
Carl	4	0	EUR 16 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 9.4 million
	0	5	EUR 9 million
Greg	4	5	EUR 22 million

The same bidders (Alan, Bob, Carl and Fred) win the same number of paired and unpaired blocks as in the previous example. This is the case because Greg is only willing to pay EUR 22 million for a package of four paired and five unpaired abstract frequency blocks, while Alan (four paired abstract frequency blocks for EUR 14 million) and Fred (5 unpaired abstract frequency blocks for EUR 9 million) are willing to pay a total of EUR 23 million for the same package. However, Greg's participation in the auction increases the potential base prices for the winning bids.

Steps a) and b): For each winning bidder, calculate the sum of the bid amounts which would result from the winner determination procedure if that bidder had not submitted a bid. Determine the minimum bids and the maximum discounts.

If Alan had not taken part in the auction, then the next best combination in terms of revenues would consist of Greg’s bid, Fred not acquiring any blocks, and the other two bidders retaining their winning bids. Thus the next highest total bid value would be EUR 59.8 million, meaning that the maximum discount for Alan would be EUR 1 million and the potential base price for Alan's winning bid would be EUR 13 million. Table 13 shows the potential base prices for each winning bid when the same process is applied to the other bidders.

Table 13: Potential base prices – Example 4

Bidder	Next best maximum revenue amount	Maximum discount	Potential base price
Alan	EUR 59.8 million	Minimum of EUR 60.8 million - EUR 59.8 million = EUR 1 million and EUR 14 million - EUR 1.6 million = EUR 12.4 million i.e., EUR 1 million	EUR 13 million
Bob	EUR 59.8 million	Minimum of EUR 60.8 million - EUR 59.8 million = EUR 1 million and EUR 21.8 million - EUR 3.2 million = EUR 18.6 million i.e., EUR 1 million	EUR 20.8 million
Carl	EUR 57.8 million	Minimum of EUR 60.8 million - EUR 57.8 million = EUR 3 million and EUR 16 million - EUR 1.6 million = EUR 14.4 million i.e., EUR 3 million	EUR 13 million
Fred	EUR 59.8 million	Minimum of EUR 60.8 million - EUR 59.8 million = EUR 1 million and EUR 9 million - EUR 1 million = 8 million i.e., EUR 1 million	EUR 8 million

Step c): We then generate a list of constraints and initialise it with the conditions that the individual discounts must not be higher than the maximum discounts calculated in the previous step, and that the overall sum of discounts over all bidders together must not be larger than the reduction in the total bid amount if the bidders had not taken part in the auction.

If the winning bidders had not taken part in the auction, Doris and Greg would have won, and the next highest revenue would be EUR 29 million, meaning that the maximum sum of discounts for all winning bidders together is EUR 31.8 million.

The constraints on this linear program are therefore as follows:

$$d_{Alan} \geq 0$$
$$d_{Bob} \geq 0$$
$$d_{Carl} \geq 0$$
$$d_{Fred} \geq 0$$
$$d_{Alan} \leq 1Mio$$
$$d_{Bob} \leq 1Mio$$
$$d_{Carl} \leq 3Mio$$
$$d_{Fred} \leq 1Mio$$
$$d_{\{Alan,Bob,Carl,Fred\}} \leq 31.8Mio$$

Step d): Maximise the sum of the individual discounts subject to these constraints.

The solution to this maximisation problem subject to the constraints is as follows:

$$d^*_{Alan} = 1Mio$$
$$d^*_{Bob} = 1Mio$$
$$d^*_{Carl} = 3Mio$$
$$d^*_{Fred} = 1Mio$$

Step e): Reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero.

Subtracting the discounts from all of the winners' bids yields the following modified package bids:

Table 14: Modified package bids, first iteration – Example 4

Bidder	**Package**		**Modified bid (after application of discount)**
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	5	0	EUR 13.8 million
	4	0	EUR 13 million
Bob	6	4	EUR 20.8 million
	6	3	EUR 19.2 million
	5	4	EUR 19 million
	5	3	EUR 18.2 million
Carl	4	0	EUR 13 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 8.4 million
	0	5	EUR 8 million
Greg	4	5	EUR 22 million

Steps f) and g): Then determine the winning bids in accordance with Rule 4.10 using the modified bid amounts, calculate the corresponding total winning bid amount, and compare them with the original winning bids using the reduced bid amounts.

We again find the combination that maximises revenues (with the modified bids), which is highlighted in blue in Table 14. The sum of bids (EUR 55.8 million) is not the same as the sum of the modified winning bids (EUR 54.8 million). Compared to the original result, Alan and Fred are eliminated from the modified winning combination of bidders.

Step h): If there are winning bidders from the original winner determination procedure who are no longer among the winning bidders given the modified bids, then expand the list of constraints to include the constraint that the sum of discounts for those bidders who are eliminated from the winning combination must not be higher than the next best combination in terms of revenues if those bidders had not taken part in the auction. Proceed to Step d).

We therefore have to find the next best combination in terms of revenues on the basis of the original bids submitted (cf. Table 12) which would have arisen if Alan and Fred had not participated in the auction. This combination would yield EUR 59.8 million (i.e., Greg would receive his package in this case). Therefore, $\sigma(Alan, Fred) = 60.8Mio - 59.8Mio = 1Mio$.

Accordingly, the maximum sum of discounts for Alan and Fred can be at most EUR 1 million. We therefore add the following constraint to our list:

$$d_{Alan} + d_{Fred} \leq 1Mio$$

Step d): Maximise the sum of the individual discounts subject to these constraints.

One possible solution to the maximisation problem given the expanded list of constraints is as follows:

$$d^*_{Alan} = 0.5Mio$$
$$d^*_{Bob} = 1Mio$$
$$d^*_{Carl} = 3Mio$$
$$d^*_{Fred} = 0.5Mio$$

This is only one possible solution, as any combination of non-negative discounts for Alan and Fred that meet the condition $d_{Alan} + d_{Fred} = 1Mio$ (and, of course, the other constraints) maximises the sum of discounts.

Step e): Reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero.

Subtracting the discounts from the winners' bids in Table 12 yields the modified winning bids listed in the table below.

Table 15: Modified package bids, second iteration – Example 4

Bidder	**Package**		**Modified bid (after application of discount)**
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	5	0	EUR 14.3 million
	4	0	EUR 13.5 million
Bob	6	4	EUR 20.8 million
	6	3	EUR 19.2 million
	5	4	EUR 19 million
	5	3	EUR 18.2 million
Carl	4	0	EUR 13 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 8.9 million
	0	5	EUR 8.5 million
Greg	4	5	EUR 22 million

Steps f) and g): Then determine the winning bids in accordance with Rule 4.10 using the modified bid amounts, calculate the corresponding total winning bid amount, and compare them with the original winning bids using the reduced bid amounts.

We again find the combination which maximises revenues (with the modified bids), which is highlighted in blue in Table 15. The total bid amount (EUR 56.3 million) is not the same as the sum of the modified winning bids (EUR 55.8 million).

Step h): If there are winning bidders from the original winner determination procedure who are no longer among the winning bidders using the modified bids, then expand the list of

constraints to include the constraint that the sum of discounts for those bidders who are eliminated from the winning combination must not be higher than the next best combination in terms of revenues if those bidders had not taken part in the auction. Proceed to Step d).

Now Carl and Fred are eliminated from the original winning combination. Therefore, we have to find the next best combination in terms of revenues from Table 12 if Carl and Fred had not participated in the auction. This combination would yield EUR 57.8 million (i.e., Greg would receive his package). Therefore, $\sigma(Carl, Fred) = 3Mio$; the maximum discount which can be granted to Carl and Fred together is EUR 3 million. We now add the following constraint to our list:

$$d_{Carl} + d_{Fred} \leq 3Mio.$$

Step d): Maximise the sum of the individual discounts subject to these constraints.

The solution to the maximization problem given the expanded list of constraints is as follows:

$$\begin{aligned} d^*_{Alan} &= 1Mio \\ d^*_{Bob} &= 1Mio \\ d^*_{Carl} &= 3Mio \\ d^*_{Fred} &= 0 \end{aligned}.$$

Steps e), f) and g): Reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero. Then determine the winning bids in accordance with Rule 4.10 using the modified bid amounts, calculate the corresponding total winning bid amount, and compare them with the original winning bids using the reduced bid amounts.

Subtracting these discounts from the winners' bids in Table 12 yields the modified winning bids indicated in the table below.

Table 16: Modified package bids, third iteration – Example 4

Bidder	Package		Modified bid (after application of discount)
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	5	0	EUR 13.8 million
	4	0	EUR 13 million
Bob	6	4	EUR 20.8 million
	6	3	EUR 19.2 million
	5	4	EUR 19 million
	5	3	EUR 18.2 million
Carl	4	0	EUR 13 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 9.4 million
	0	5	EUR 9 million
Greg	4	5	EUR 22 million

The winning combination now remains the same, that is, Step f) does not yield higher revenues than the modified bids. The next step is the quadratic optimization.

Steps i), j), k) and l): Minimise the sum of squared differences between the individual discounts and the individual maximum discounts subject to the constraints and the condition that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts. Reduce the bid amounts for all of the winners' bids by their corresponding discounts; if this yields a negative bid amount, set this bid amount to zero. Determine the winning bids in accordance with Rule 4.10 using the modified bid amounts and calculate the corresponding total winning bid amount. If the original winning bids (at the reduced bid amounts) do not yield a lower value than the total bid amount calculated in Step k), then the discounts found are the relevant discounts and the base prices are equal to the winning bids reduced by those amounts.

The quadratic optimisation yields the discounts found at the end of the linear optimisation. Those discounts do not change the result of the winner determination procedure and are thus the discounts with which the corresponding base prices can be calculated. The base prices are listed in Table 17.

Table 17: Base prices for winning bids – Example 4

Bidder	Base price
Alan	EUR 13 million
Bob	EUR 20.8 million
Carl	EUR 13 million
Fred	EUR 9 million
Total:	EUR 55.8 million

The intuitive explanation for these base prices is as follows:

If – as in the previous example – the winning bidders were granted their respective maximum individual discounts, then Greg's bid would be higher than the sum of Fred and Alan's or Fred and Carl's modified winning bids, and thus it would maximise revenues if Greg's bid were accepted instead of Fred's and Alan's. This gives rise to additional constraints on the base prices:

- Taken together, Alan and Fred's base prices must be higher than Greg's bid for this package;
- Taken together, Carl and Fred's base prices must be higher than Greg's bid for this package.

This means that the total of base prices for Alan and Fred and for Carl and Fred must in each case be EUR 1 million higher than the sum of the individual opportunity costs for those bidders.

As the sum of all base prices is to be minimised (and thus the sum of all discounts maximised), there is only one solution in this example: Fred must pay the additional required amount of EUR 1 million on his own. Otherwise both Alan and Carl would pay a higher base price, and the sum of base prices would thus be higher than if Fred alone pays the additional amount. If, for example, Fred only pays half of the additional EUR 1 million, then both Alan and Carl would each have to pay an additional EUR 0.5 million, and the sum of base prices would be EUR 56.3 million. This is higher than the sum of base prices in the scenario where Fred alone pays the EUR 1 million (EUR 55.8 million).

Example 5: Calculation of base prices – Minimisation of distance from individual opportunity costs

Example 4 illustrated a case in which a group of winning bidders must pay an amount which is larger than the sum of individual opportunity costs in order to ensure that the combination of bids which maximises revenues remains unchanged.

In the previous example, the rule that the sum of discounts should be maximised (or, conversely, that the sum of base prices for all winning bids should be minimised) was sufficient in order to identify a unique combination of base prices. However, this is not always the case.

In the example below, we combine Carl's and Alan's demand (i.e., we assume that Alan now bids on eight paired abstract frequency blocks) and we double Greg's demand for paired spectrum (Greg now bids on eight paired and five unpaired abstract frequency blocks). The corresponding bids are summarised in Table 18. The winning combination of bids is again highlighted in blue.

Table 18: Combinatorial package bids in the principal stage – Example 5

Bidder	Package		Package bid (principal stage)
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	8	0	EUR 30 million
Bob	6	4	EUR 21.8 million
	6	3	EUR 20.2 million
	5	4	EUR 20 million
	5	3	EUR 19.2 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 9.4 million
	0	5	EUR 9 million
Greg	8	5	EUR 35 million

Steps a) and b): For each winning bidder, calculate the sum of the bid amounts which would result from the winner determination procedure if that bidder had not submitted a bid. Determine the minimum bids and the maximum discounts.

The maximum possible discounts are again calculated as described in the previous examples. Table 19 provides a summary of the maximum discounts and the potential base prices.

Table 19: Potential base prices – Example 5

Bidder	Next best maximum revenue amount	Maximum discount	Potential base price
Alan	EUR 56.8 million	Minimum of EUR 60.8 million - EUR 56.8 million = EUR 4 million and EUR 30 million - EUR 3.2 million = EUR 26.8 million i.e., EUR 4 million	EUR 26 million
Bob	EUR 46 million	Minimum of EUR 60.8 million - EUR 46 million = EUR 14.8 million and EUR 21.8 million - EUR 3.2 million = EUR 18.6 million i.e., EUR 14.8 million	EUR 7 million
Fred	EUR 59.8 million	Minimum of EUR 60.8 million - EUR 59.8 million = EUR 1 million and EUR 9 million - EUR 1 million = EUR 8 million i.e., EUR 1 million	EUR 8 million

Step c): We then generate a list of constraints and initialise it with the conditions that the individual discounts must not be higher than the maximum discounts calculated in the previous step, and that the overall sum of discounts over all bidders together must not be larger than the reduction in the total bid amount if the bidders had not taken part in the auction.

If the winning bidders had not taken part in the auction, then the next best combination in terms of maximum revenues would be to allow Greg and Doris to win their respective package bids. The revenues from this combination would be EUR 42 million; the sum of all discounts over all winners is thus EUR 18.8 million (EUR 60.8 million – EUR 42 million).

The constraints result from Table 16 and are as follows:

$$d_{Alan} \geq 0$$
$$d_{Bob} \geq 0$$
$$d_{Fred} \geq 0$$
$$d_{Alan} \leq 4 Mio$$
$$d_{Bob} \leq 14{,}8 Mio$$
$$d_{Fred} \leq 1 Mio$$
$$d_{\{Alan,Bob,Fred\}} \leq 18{,}8 Mio$$

Step d): Maximise the sum of the individual discounts subject to these constraints.

Maximising the sum of discounts under the given constraints yields, inter alia, the following solution:[8]

$$d^*_{Alan} = 4\,Mio$$
$$d^*_{Bob} = 14.8\,Mio$$
$$d^*_{Fred} = 0\,Mio$$

Steps e), f) and g): Reduce the bid amounts for all winning bidders by the discounts found; if this yields a negative bid amount, set this bid amount to zero. Then determine the winning bids in accordance with Rule 4.10 using the modified bid amounts, calculate the corresponding total winning bid amount, and compare them with the original winning bids using the reduced bid amounts.

Subtracting the discounts from the winners' bids in Table 18 yields the modified bids shown in Table 20.

[8] This is only one possible solution; other combinations of discounts also meet the ancillary conditions. The solution identified by the algorithm is independent of the specific combination selected in this step.

Table 20: Modified bids, first iteration – Example 5

Bidder	Package		Bid in principal stage
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	8	0	EUR 26 million
Bob	6	4	EUR 7 million
	6	3	EUR 5.4 million
	5	4	EUR 5.2 million
	5	3	EUR 4.4 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 9.4 million
	0	5	EUR 9 million
Greg	8	5	EUR 35 million

It turns out that the amount of the revenue-maximising combination of modified bids, which yields EUR 42 million, (Step f), is not higher than the sum of the original modified winning bids (i.e., the modified bids for Alan, Bob and Fred).

Therefore, the original winning combination of bidders remains optimal (even if another combination – i.e., the combination of Bob and Greg's bids – yields the same value), and we can thus proceed to the quadratic optimisation.

Steps i), j), and k): Minimise the sum of squared differences between the individual discounts and the individual maximum discounts subject to the constraints and the condition that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts. Reduce the bid amounts for all of the winners' bids by their corresponding discounts; if this yields a negative bid amount, set this bid amount to zero. Determine the winning bids in accordance with Rule 4.10 using the modified bid amounts and calculate the corresponding total winning bid amount.

The solution to the quadratic minimisation program subject to the constraints is as follows:

$$d^{**}_{Alan} = 3.66...Mio$$

$$d^{**}_{Bob} = 14.466...Mio$$

$$d^{**}_{Fred} = 0.66...Mio$$

We reduce all of the winners' bids by the discounts found, to obtain the modified bids provided in Table 21.

Table 21: Modified bids, second iteration – Example 5

Bidder	Package		Modified bid (after application of discount)
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	8	0	EUR 26.33... million
Bob	6	4	EUR 7.33... million
	6	3	EUR 5.73... million
	5	4	EUR 5.53... million
	5	3	EUR 4.73... million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 8.73... million
	0	5	EUR 8.33... million
Greg	8	5	EUR 35 million

Step l): Check whether the original winning bids – using the reduced bid amounts – yield a lower value than the total bid amount calculated in Step k).

With the winning bids modified in this way, Alan and Fred would no longer be among the winning bidders: The quadratic optimisation shows that the difference between the sum of the maximum individual discounts (EUR 19.8 million) and the maximum discounts for all winning bidders (EUR 18.8 million) is to be distributed equally among all winning bidders. However, this would make Alan's and Fred's discounts too high, and Bob's discount would be too low. The combination of Bob and Greg would thus be superior to the original winning combination of bidders.

Step m): If there are winning bidders from the original winner determination procedure who are no longer among the winning bidders using the modified bids, then expand the list of constraints to include the constraint that the sum of discounts for those bidders who are eliminated from the winning combination must not be higher than the next best combination in terms of revenues if those bidders had not taken part in the auction. Proceed to Step i).

On the basis of the bids in Table 18, we calculate the maximum sum of discounts granted to Alan and Fred to be $\sigma(Alan, Fred) = 4Mio$, which is the difference between the maximum total revenues from the original bids and the revenues from the next best combination if Alan and Fred had not taken part in the auction. This implies that the sum of discounts for Alan and Fred cannot be higher than EUR 4 million. We therefore add the following constraint to the list of constraints:

$$d_{Alan} + d_{Fred} \leq 4 .$$

The solution to the minimisation problem under the expanded list constraints is as follows:

$$d^{**}_{Alan} = 3.5Mio$$
$$d^{**}_{Bob} = 14.8Mio$$
$$d^{**}_{Fred} = 0.5$$

We then reduce all of the winners' bids by the discounts found, thus obtaining the bids shown in Table 21. We then proceed to Steps i) to k).

Table 22: Modified bids, third iteration – Example 5

Bidder	Package		Modified bid (after application of discount)
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	8	0	EUR 26.5 million
Bob	6	4	EUR 7 million
	6	3	EUR 5.4 million
	5	4	EUR 5.2 million
	5	3	EUR 4.4 million
Doris	0	4	EUR 7 million
Emma	0	5	EUR 8 million
Fred	0	6	EUR 8.9 million
	0	5	EUR 8.5 million
Greg	8	5	EUR 35 million

Step l): If the original winning bids (at the reduced bid amounts) do not yield a lower value than the total bid amount calculated in Step k), then the discounts found are the relevant discounts and the base prices are equal to the winning bids reduced by those amounts.

The resulting base prices are shown in Table 23.

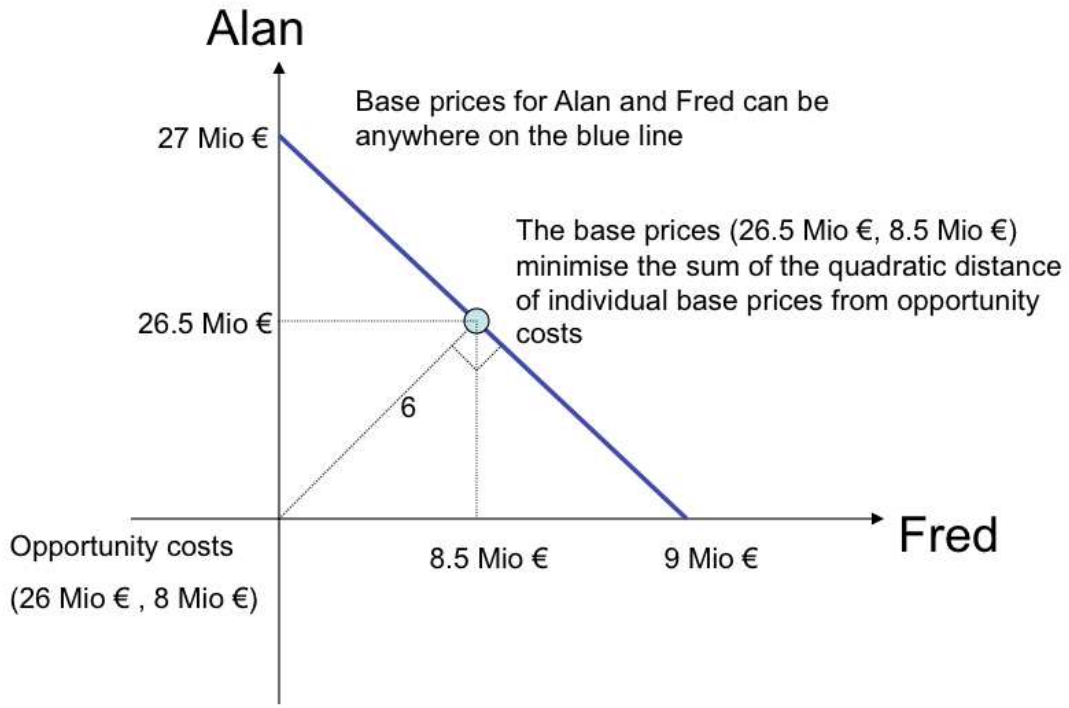
Table 23: Base prices for winning bids – Example 5

	Base price
Alan	EUR 26.5 million
Bob	EUR 7 million
Fred	EUR 8.5 million

The intuitive explanation for this solution is as follows:

As in Example 4, the sum of the base prices for Fred and Alan has to be high enough to outbid Greg. In this example, the sum of the opportunity costs for Alan and Fred's packages of eight paired abstract frequency blocks (EUR 26 million) and five unpaired abstract frequency blocks (EUR 8 million) is equal to EUR 34 million. However, this sum is lower than what Greg would be willing to pay for this combination (EUR 35 million). Fred and Alan thus have to pay the additional EUR 1 million together. There are various ways to split this amount between them. The figure below shows all combinations of base prices for Alan and Fred which (taken together) are as high as Greg's bid (blue line in Figure 2).

Figure 2: Minimisation of squared differences – Example 5



In order to determine a unique set of base prices, the sum of squared distances of base prices from opportunity costs is minimised. This selection criterion always identifies a unique set of base prices.

Appendix C: Determination of assignment bid options

The auction software provides the bidders with all possible assignment bid options. These options must meet the conditions set out in 5.1.

Algorithmic determination of assignment options

The possible assignment options can be determined algorithmically as follows:

Let W be the set of bidders who acquired frequency blocks (i.e., lots) in the principal stage.

Let L_i^k be the number of blocks in category k acquired by bidder $i \in W$.

- Generate a list P^k of all possible arrangements of bidders in category k. Such an arrangement is an ordered list of the winning bidders according to which the bidders are assigned the frequency blocks won in the principal stage (starting from the lowest block which can be assigned in the category) in the sequence in which they are listed.
- For every possible arrangement $p^k \in P^k$, a valid assignment option for bidder i is determined as the bid on the (starting) frequency block $n = 1 + \sum_{j=1}^{\omega_i(p^k)} L_j^k - L_i^k$ (and the following $L_i^k - 1$ blocks), with $\omega_i(p^k)$ denoting the position of bidder i in the arrangement p^k and *n* denoting the position of the starting block in the spectrum range to be assigned (cf. the rules defined under 5.1). Note: Different possible arrangements may result in the same assignment option for a given bidder.

Usually, the assignment bid options should be available in less than one second. In certain situations (e.g., more than ten winning bidders in the paired range), this process may take significantly longer, or additional processing capacity may be required. In such an improbable case, delays may arise in the assignment process.

Two examples are given below to illustrate how these options are determined. In Example 6, the entire 2.6 GHz band is awarded, while Example 7 presents a case in which frequency blocks remain unsold.

Example 6: Determination of assignment options

In this example, all abstract frequency blocks were auctioned off in the principal stage, with the following winners emerging from that stage:

- Alan won four paired abstract frequency blocks and three unpaired abstract frequency blocks;
- Ben won four paired abstract frequency blocks;
- Carl won six paired abstract frequency blocks;
- Dana won six unpaired abstract frequency blocks.

The assignment stage is subdivided into two simultaneous procedures:

- the auction for actual paired frequency blocks;
- the auction for actual unpaired frequency blocks.

The possible assignment options for paired spectrum are shown in Figure 3. The possible assignment options for each bidder are as follows:

- Alan: A1-A4; A5-A8; A7-A10; A11-A14.

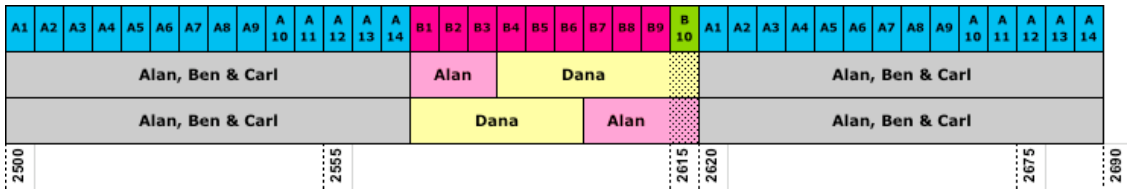
- Ben: A1-A4; A5-A8; A7-A10; A11-A14.
- Carl: A1-A6; A5-A10; A9-A14.

Figure 3: Assignment options for paired spectrum – Example 6

The possible assignment options for unpaired spectrum are shown in Figure 4. The possible assignment options for each bidder are as follows:

- Alan: B1-B3; B7-B9 (with the additional frequency block B10).
- Dana: B1-B6; B4-B9 (with the additional frequency block B10).

Figure 4: Assignment options for unpaired spectrum – Example 6



Example 7: Determination of assignment options in the case of unsold frequency blocks

In this example, not all of the abstract frequency blocks were acquired in the principal stage. A total of two paired abstract frequency blocks and one unpaired abstract frequency block remained unsold in the principal stage. As discussed in Rule 5.1.3, the paired spectrum is to be assigned in contiguous blocks from 2500 MHz upward, and the unpaired spectrum is to be assigned in contiguous blocks from 2615 MHz downward in this case. This means that the frequency blocks A13, A14 and B1 will not be assigned in this example. The following bidders acquired abstract frequency blocks in the principal stage:

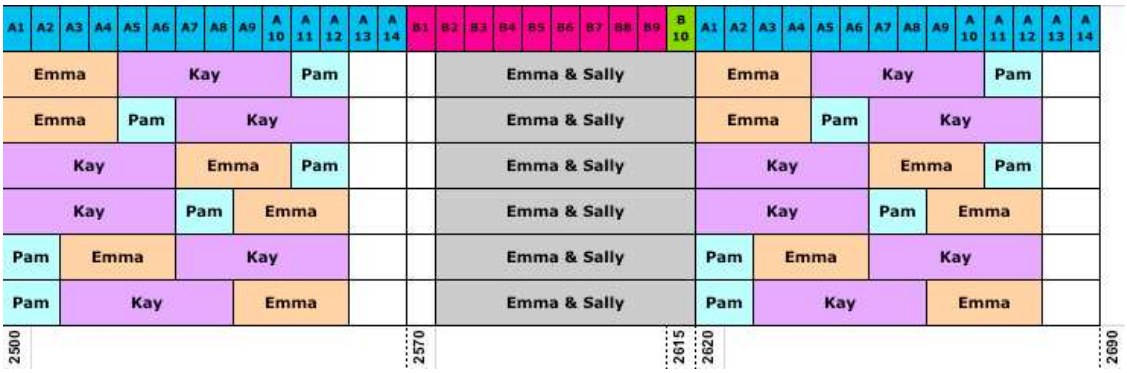
- Emma won four paired abstract frequency blocks and three unpaired abstract frequency blocks;
- Kay won six paired abstract frequency blocks;
- Pam won two paired abstract frequency blocks;
- Sally won five unpaired abstract frequency blocks.

The possible assignment options for paired spectrum are shown in Figure 5. The possible assignment options for each bidder are as follows:

- Emma: A1-A4; A3-A6; A7-A10; A9-A12.
- Kay: A1-A6; A3-A8; A5-A10; A7-A12.

- Pam: A1-A2; A5-A6; A7-A8; A11-A12.

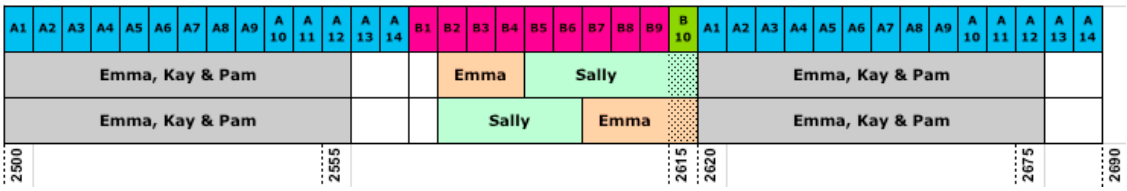
Figure 5: Assignment options for paired spectrum – Example 7



The possible assignment options for unpaired spectrum are shown in Figure 6. The possible assignment options for each bidder are as follows:

- Emma: B2-B4; B7-B9 (with the additional frequency block B10).
- Sally: B2-B6; B5-B9 (with the additional frequency block B10).

Figure 6: Assignment options for unpaired spectrum – Example 7



Appendix D: Calculation of top-up prices

Description of the algorithm used to calculate top-up prices in accordance with Rule 5.5.

The procedure described below finds a unique set of top-up prices which meet the conditions of Rule 5.5. The procedure is essentially similar to the one used to calculate base prices, except for the fact that exactly one assignment bid from each bidder must be accepted.

Let W be the set of bidders participating in the assignment stage (i.e., the bidders who placed winning bids in the principal stage). The winner determination procedure in accordance with Rule 5.4 determines the sum of the winning bids v and the assignment option valid for each bidder. v^{-C} denotes the sum of the winning assignment bids identified in the winner determination procedure if the bidders in $C \subseteq W$ do not express any preferences for the assignment of certain blocks, that is, if all of the assignment bids submitted by these bidders show a bid amount of zero. $\sigma(C) = (v - v^{-C})$ denotes the decline in the total assignment bid value if the bidders in $C \subseteq W$ do not submit any positive assignment bids.

a) Generate a list of constraints B for a restricted optimisation problem and initialise it as $B = \left\{ \forall j \in W : d_j \leq \sigma(\{j\}), \sum_{j=1}^{J} d_j \leq \sigma(W) \right\}$.

b) Solve the following maximisation problem: $\max \sum_{j \in W} d_j$ subject to the constraints in B and the constraint that the individual values of d_j cannot be negative. In general, many solutions to this maximisation problem may exist. If this is the case, pick a random solution d^*.

c) Reduce the bid amounts for all assignment bids submitted by Bidder j by d_j^*; if this yields a negative bid amount, set this bid amount to zero.

d) Determine the winning assignment bids in accordance with Rule 5.4 using the modified bid amounts and calculate the corresponding total winning bid amount.

e) If the original winning assignment bids (at the reduced bid amounts) do not yield a lower value than the total bid amount calculated in Step d), then the maximum discount which can be granted to all winning assignment bidders is $D^* = \sum_{j \in W} d_j^*$.

Proceed to Step g).

f) If there are winning assignment bidders from the original assignment winner determination procedure whose assignment option given the modified assignment bids is not the same as the option resulting from the original assignment bids and whose modified assignment bid is not positive,[9] then add the following constraint to the list of constraints in B:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

[9] This reflects the fact that the "losers" are those bidders whose preferences for certain assignment options are not taken into account. This means that where bidders receive a different assignment option with a modified assignment bid of zero compared to their original assignment bid, those bidders were granted too large a discount.

where L denotes the set of bidders who receive a different assignment option and whose modified assignment bid is not positive. Proceed to Step b).

g) Solve the following minimisation problem: $\min \sum_{j \in W} \left(d_j - \sigma(\{j\})\right)^2$ subject to the constraints in B, the constraint that the individual values of d_j cannot be negative, and the constraint that the sum of the individual discounts must be equal to the maximum permissible overall sum of discounts, i.e., $\sum_{j \in W} d_j = D^*$. Let d^{**} be the solution to this minimisation problem.

h) Reduce the bid amounts for all assignment bids submitted by Bidder j by d_j^{**}; if this yields a negative bid amount, set this bid amount to zero.

i) Determine the winning assignment bids in accordance with Rule 5.4 using the modified assignment bid amounts and calculate the corresponding total winning bid amount.

j) If the original winning assignment bids (at the reduced bid amounts) do not yield a lower value than the total bid amount calculated in Step i), then the values of d_j^{**} are the relevant discounts and the top-up prices are equal to the winning assignment bids reduced by these amounts, that is, $p_j = b_j - d_j^{**}$ for all $j \in W$, with b_j denoting the bidder's winning assignment bid.

k) If there are winning assignment bidders from the original assignment winner determination procedure whose assignment option given the modified assignment bids is not the same as the option resulting from the original assignment bids and whose modified assignment bid is not positive, then add the following constraint to the list of constraints in B:

$$\sum_{j \in L} d_j \leq \sigma(L)$$

where L denotes the set of bidders who receive a different assignment option and whose modified assignment bid is not positive. Proceed to Step g).

Due to numerical inaccuracies in the automated calculation of top-up prices (minimum revenue core prices), minimal deviations from the theoretical top-up price may arise; however, as figures will be rounded up to the next whole euro amount, these deviations will generally not have any effect on the calculated top-up price. In very rare cases, the top-up price calculated may be a few euros higher than the theoretical top-up price.

Usually, the results should be available in less than one second. Due to the inherent complexity of the combinatorial clock auction, in certain situations it may take significantly longer to calculate the results, or additional processing capacity may be required. In such an improbable case, the submission of bids will be concluded and the results will be announced at a later point in time.

Example 8: Calculation of top-up prices

This example is based on Example 3 (calculation of base prices where base prices are equal to opportunity costs). In the principal stage, the winning bidders and bids listed in Table 24 are determined.

Table 24: Winning bidders and winning bids – Example 8

Bidder	Package		Base price
	Number of paired abstract frequency blocks	Number of unpaired abstract frequency blocks	
Alan	4	0	EUR 1.6 million
Bob	6	4	EUR 7.8 million
Carl	4	0	EUR 1.6 million
Fred	0	5	EUR 8 million

Figure 7 shows all possible options for the assignment of paired spectrum in accordance with the rules set out in 5.1. The following assignment options are available to the bidders:

- Alan: A1-A4; A5-A8; A7-A10; A11-A14.
- Bob: A1-A6; A5-A10; A9-A14.
- Carl: A1-A4, A5-A8; A7-A10; A11-A14.

Figure 7: Assignment options for paired spectrum – Example 8

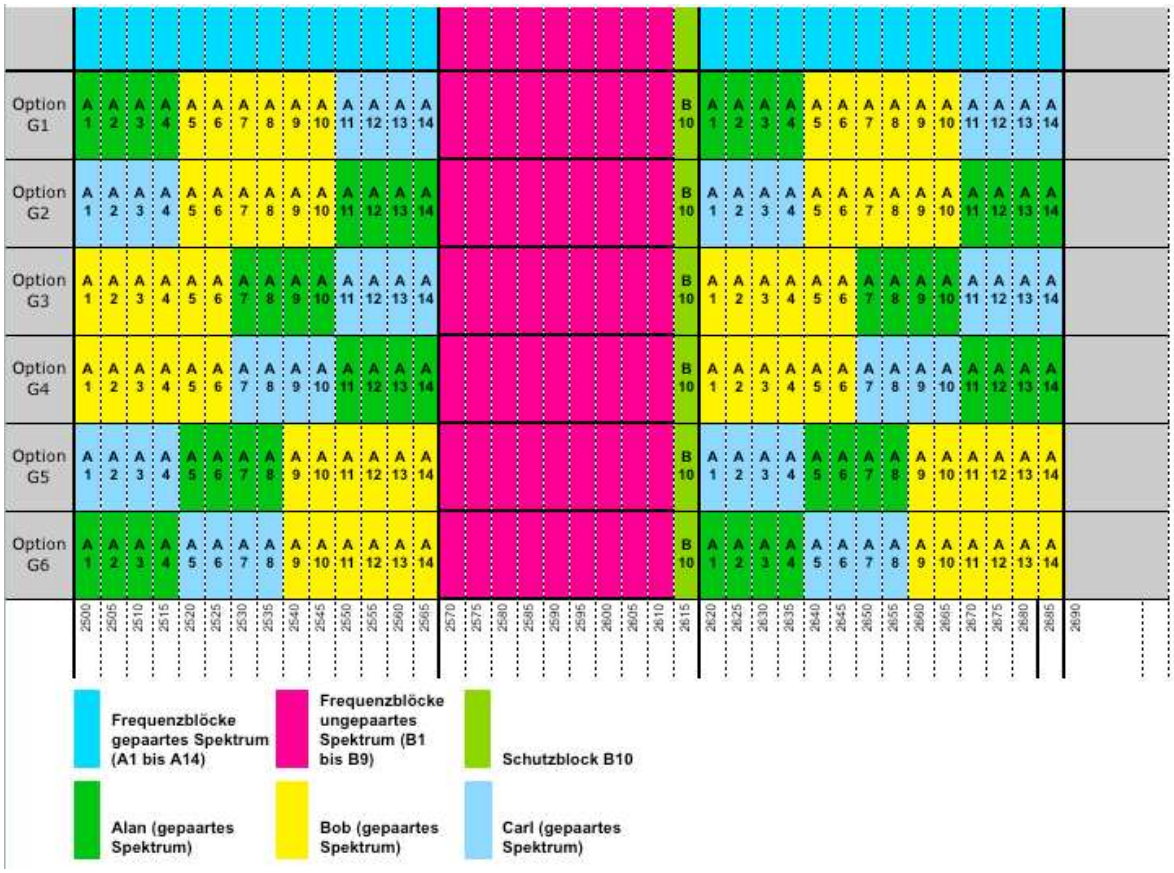
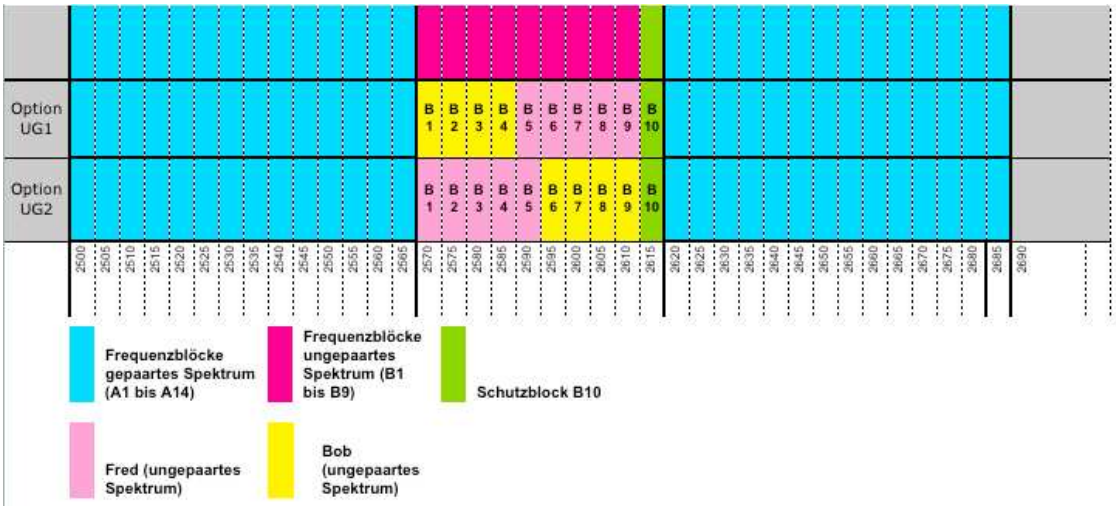


Figure 8 shows all possible options for the assignment of unpaired spectrum in accordance with the rules defined under 5.1. The following assignment options are available to the bidders:

- Bob: B1-B4; B6-B9 (including B10).

- Fred: B1-B5; B5-B9 (including B10).

Figure 8: Assignment options for unpaired spectrum – Example 8



During the assignment stage, the bidders submit the assignment bids shown in Table 25 (zero bids are submitted automatically by the auction software).

Table 25: Assignment bids – Example 8

Bidder	Number of abstract frequency blocks acquired	Category	Starting block of assignment option	Assignment bid	Options
Bob	6	Paired	A9	EUR 0.5 million	G5, G6
Bob	6	Paired	A1	EUR 0 million	G3, G4
Bob	6	Paired	A5	EUR 0 million	G1, G2
Alan	4	Paired	A1	EUR 1 million	G1, G6
Alan	4	Paired	A7	EUR 0.3 million	G3
Alan	4	Paired	A5	EUR 0 million	G5
Alan	4	Paired	A11	EUR 0 million	G2, G4
Carl	4	Paired	A1	EUR 0.8 million	G2, G5
Carl	4	Paired	A11	EUR 0.9 million	G1, G3
Carl	4	Paired	A5	EUR 0 million	G6
Carl	4	Paired	A7	EUR 0 million	G4
Bob	4	Unpaired	B1	EUR 0.1 million	UG1
Bob	4	Unpaired	B6	EUR 0 million	UG2
Fred	5	Unpaired	B5	EUR 0.3 million	UG1
Fred	5	Unpaired	B1	EUR 0 million	UG2

The winning combination of bids for each category is highlighted in blue.

As in the winner determination procedure in the principal stage, the highest combination of assignment bids is determined under the conditions specified in Rule 5.4. The total bids for the various assignment options for paired spectrum are as follows:

- G1: EUR 1.9 million;
- G2: EUR 0.8 million;
- G3: EUR 1.2 million;
- G4: EUR 0;
- G5: EUR 1.3 million;
- G6: EUR 1.5 million.

In the paired spectrum, the winning bids are Carl's assignment bid of EUR 0.9 million for frequency blocks A11 to A14 and Alan's assignment bid of EUR 1 million for frequency blocks A1 to A4. Therefore, Option G1 is the winning combination. Bob receives frequency blocks A5 to A10.

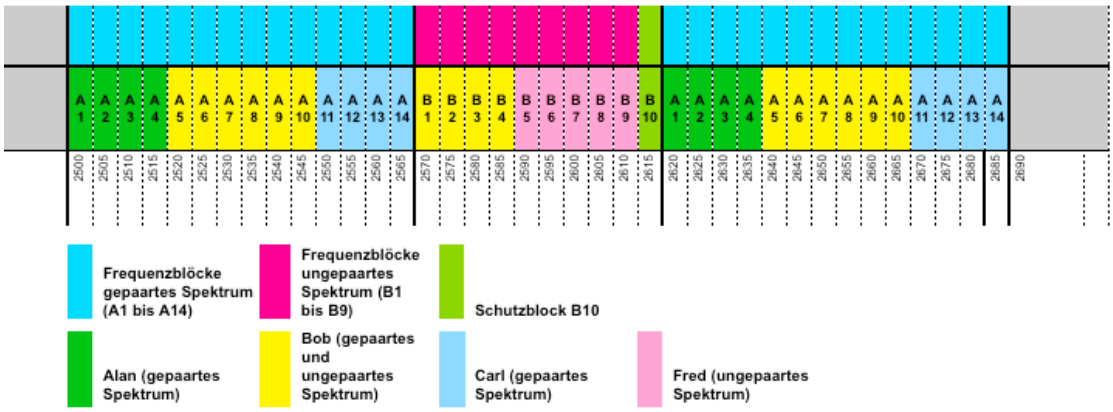
The total bids for the various assignment options for unpaired spectrum are as follows:

- UG1: EUR 0.4 million;
- UG1: EUR 0.

In the unpaired spectrum, Option UG1 is the winning combination for the assignment of frequency blocks.

Figure 9 shows the winning combinations of bids that maximise revenue and meet the conditions under Rule 5.4.

Figure 9: Winning bids in the assignment stage – Example 8



The calculation of top-up prices using the modified second-price rule is similar to the calculation of base prices in the principal stage. The difference is that in the calculation of opportunity costs (or the maximum discounts according to Step a)), not all of a bidder's assignment bids are eliminated; instead, the bid amounts are set to the respective minimum bid (in this case zero). The reason for this difference is that the opportunity costs are determined by a bidder's preference for certain actual frequency blocks and not by the bidder's demand for frequency blocks in general. The reduction of all bids to the respective minimum bids reflects the case in which a bidder expresses no preferences for specific frequency blocks.

In the paired spectrum, the winning bids are the assignment bids submitted by Carl, Alan and Bob in Option G1 (a zero bid was placed automatically for Bob). If one were to set all of Carl's assignment bids to zero, Option G6 would win instead of Option G1. Therefore, if Carl did not have any preference for a specific assignment option, then Bob would win blocks A9 to A14 and Alan would receive blocks A1 to A4. The revenues would then be (EUR 1 million + EUR 0.5 million = EUR 1.5 million). This means that the maximum discount (according to Step a)) for Carl is EUR 0.4 million and the opportunity costs arising from Carl's preferences

are EUR 0.5 million (EUR 0.9 million - EUR 0.4 million). An analogous procedure is carried out for the other bidders. The results are summarised in the table below.

Table 26: Calculation of top-up prices – Example 8

Bidder	Frequency blocks	Modified maximum revenue amount	Maximum discount	Opportunity costs (potential top-up price)
Alan	A1-A4	EUR 1.3 million	EUR 1.9 million - EUR 1.3 million = EUR 0.6 million	EUR 1 million - EUR 0.6 million = EUR 0.4 million
Bob	A5-A10;	EUR 1.9 million	EUR 1.9 million - 1.9 million = EUR 0;	EUR 0 - EUR 0 = EUR 0;
	B1-B4	EUR 0.3 million	EUR 0.4 million - EUR 0.3 million = EUR 0.1 million	EUR 0.1 million - EUR 0.1 million = EUR 0
Carl	A11-A14	EUR 1.5 million	EUR 1.9 million - EUR 1.5 million = EUR 0.4 million	EUR 0.9 million - EUR 0.4 million = EUR 0.5 million
Fred	B5-B9	EUR 0.1 million	EUR 0.4 million - EUR 0.1 million = EUR 0.3 million	EUR 0.3 million - EUR 0.3 million = EUR 0

The linear optimisation (Step b)) yields discounts in the maximum amounts according to Table 26. Subtracting the discounts from the bids yields the modified assignment bids shown in Table 27 (Step c). The original combination of winning bidders still maximises the revenues from the auction (Steps d) and e)), that is, the top-up prices are equal to the opportunity costs. This is illustrated in the table below.

Table 27: Modified assignment bids – Example 8

Bidder	Number of abstract frequency blocks acquired	Category	Starting block of assignment option	Modified assignment bid	Options
Bob	6	Paired	A9	0.5 million	G5, G6
Bob	6	Paired	A1	EUR 0	G3, G4
Bob	6	Paired	A5	EUR 0	G1, G2
Alan	4	Paired	A1	0.4 million	G1, G6
Alan	4	Paired	A7	EUR 0	G3
Alan	4	Paired	A5	EUR 0	G5
Alan	4	Paired	A11	EUR 0	G2, G4
Carl	4	Paired	A1	0.4 million	G2, G5
Carl	4	Paired	A11	0.5 million	G1, G3
Carl	4	Paired	A5	EUR 0	G6
Carl	4	Paired	A7	EUR 0	G4

Bob	4	Unpaired	B1	EUR 0	UG1
Bob	4	Unpaired	B6	EUR 0	UG2
Fred	5	Unpaired	B5	EUR 0	UG1
Fred	5	Unpaired	B1	EUR 0	UG2

Adding the base prices determined in the principal stage (cf. Table 24) and the top-up prices determined in the assignment stage yields the overall price for each winning bidder (cf. Rule 5.6). The results of the auction are shown in Table 28.

Table 28: Auction results – Example 8

Bidder	Package acquired	Category	Base price	Top-up price	Overall price
Bob	A5-A10	paired	EUR 7.8 million	EUR 0	EUR 7.8 million
	B1-B4	unpaired		EUR 0	
Alan	A1-A4	paired	EUR 1.6 million	EUR 0.4 million	EUR 2 million
Carl	A11-A14	paired	EUR 1.6 million	EUR 0.5 million	EUR 2.1 million
Fred	B5-B9	unpaired	EUR 8 million	EUR 0	EUR 8 million

Appendix E: Algorithm for determining winners

The purpose of the winner determination procedure is to identify the combination of bids which yields the highest total bid amount, subject to accepting at most one bid per bidder (in the assignment stage: exactly one bid per bidder) and that no more lots (= frequency blocks) are assigned than are available.

The approach we use is based on the insight that any optimal distribution of the available lots among the winning bidders must distribute subsets of lots optimally among subsets of winning bidders. This allows us to split the winner determination procedure into sub-problems and to solve these sequentially. One suitable approach for this purpose is as follows:

- Identify the set T of all possible subsets of lot combinations given the overall set of lots L.
- For each lot combination $t \in T$, identify the maximum revenues $v_1(t)$ which could be generated if the first bidder were assigned the lot combination t; the first bidder may be chosen at random.
- For the remaining $n = 2,...,N$ bidders: For each lot combination $t \in T$, identify the maximum revenues $v_n(t)$ which could be generated if the first n bidders were assigned the lot combination t; this value is derived from the optimal assignment for Bidder n in combination with the maximum value which can be generated when the remaining lots $(t-b)$ are assigned to the remaining $n-1$ bidders (i.e., $v_{n-1}(t-b)$, where b is the optimal lot combination assigned to Bidder n.
- The maximum total revenues then amount to $v_N(L)$.
- Determine the optimal bid b_N^* contained in $v_N(L)$.
- For the remaining $n = N-1,...,1$ bidders: Identify the optimal assignment for Bidder n using $v_n(L - \sum_{i=n+1}^{N} b_i^*)$.